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SDAC MSD
Co-optimization Roadmap Study: Explanatory note

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This document aims at providing additional context & background to the results of the Co-optimization roadmap study performed by N-SIDE based on input by the TSOs and NEMOs. It complements the technical material which was provided by N-SIDE (pdf slide-set and final report). The document especially targets parties not being involved in the technical discussions for which additional information may be useful (e.g., SDAC MCSC, NRAs, ACER, EC, other TSOs and NEMOs' representatives).

Disclaimer:

- Through this research, NEMOs and TSOs together with N-SIDE investigated co-optimization implementation options as identified by the ENTSO-E project team and NEMOs. N-SIDE was asked to develop a prototype model of the Cross Zonal Capacity Allocation Optimization Function (CZCAOF) and analyse and compare **the timeline, accuracy, and costs** for each possible option. Before implementing co-optimization, a high-level analysis needs to be complemented by an in-depth analysis of alternative implementation options based on a prototype analysis.

CONTEXT

EB Regulation Art. 40 describes a methodology for a co-optimised allocation process of CZC for the exchange of balancing capacity or sharing of reserves and cross-border matching of Day-Ahead Market (DAM) bids, so-called co-optimisation. Co-optimisation requires the central optimisation of the allocation of CZC based on actual DAM bids and actual Balancing Capacity Market (BCM) bids.

Following ACER's Decision 12/2020[1], a price coupling algorithm shall incorporate a cross-zonal capacity allocation optimisation function which shall optimize the allocation of cross-zonal capacity between SDAC and the exchange of balancing capacity or sharing of reserves. For this purpose, a set of requirements has been defined by mid-2022. In order to elaborate these requirements, TSOs and NEMOs performed a technical feasibility analysis of the options identified for co-optimization. Providing a technical feasibility analysis for co-optimization requires a model-based comparison of co-optimization implementation options. TSOs and NEMOs approached N-SIDE to further elaborate on co-optimisation implementation options as identified by the ENTSO-E project team and NEMOs. This prototype analysis provides a first overview with preliminary information about costs and accuracy of the different co-optimization options for NRAs/ACER.

OBJECTIVE AND ASSUMPTIONS

The main objective of the presented co-optimization study is to provide more insights into the implement ability of co-optimized allocation process of the balancing capacity market and day-ahead energy market, mainly comparing:

- 1-step vs. a 2-step implementation of co-optimized CZC allocation and
- multilateral vs. unilateral cross-product linking of bids between Balancing Capacity Markets (BCMs) and the Day Ahead Market (DAM).

This was the very first data-based study employing a potential combination of Cross-Zonal Capacity Allocation Optimization Function (CZCAOF), Single Day-ahead Market Coupling (SDAC) and Capacity Procurement Optimization Function (CPOF) under the conditions of co-optimized CZC allocation. The prototype study should identify expected operational obstacles in terms of optimization complexity and calculation time. As a quick

[1] Available on:
https://extranet.acer.europa.eu/Official_documents/Acts_of_the_Agency/Annexes%20to%20the%20DECISION%20OF%20THE%20AGENCY%20FOR%20THE%20C11/ACER%20Decision%20on%20CO%20CZCA%20-Annex%20I.pdf

and easy-to-implement approach N-SIDE enhanced Euphemia by expanding the existing SDAC optimization algorithm by the co-optimized allocation requirements (CZCAOF & CPOF) for energy and balancing capacity.

The following basic assumptions apply to the model used to obtain the results:

Starting model Three scenarios were requested by ENTSO-E to be implemented to embody the co-optimization of energy and reserve. In every scenario, the three following functions are realized either simultaneously in the same optimization program or sequentially: - CZC split: the split and allocation of part of the cross-zonal capacity to either the balancing capacity or the day-ahead energy market. - BCM clearing: clearing of the balancing capacity market - DAM clearing: clearing of the day-ahead electricity market Each scenario is given by the combination of two aspects: the linking of bids and the number of steps taken. The linking of bids between the day-ahead and the balancing capacity markets was analyzed as part of the co-optimization of energy and reserve capacity investigation. The purpose of bid linking in a co-optimization of energy and reserve capacity context is to allow market participants to better express their technical and economic characteristics while being able to bid in both markets. Each scenario considers only one type of bid linking namely multilateral or unilateral. In addition to the considered linking, each scenario contains a procedure with one or two steps. The difference between one step and two steps lies in the use of Euphemia in a second stage to clear the day-ahead energy market as it is done in Europe currently. These three scenarios consist in: - Scenario A1. One-step multilateral linking: Full co-optimization in a single optimization process with multilateral linking of bids in which the deterministic requirement (i.e., that the network can cope always with any TSO demand activated in real time) is taken into account through the inscribed boxes method. - Scenario A2. One-step unilateral linking: A sequential optimization process with first an attempt clearing of the balancing capacity market (without taking the day-ahead market into account to enforce prioritization of linked balancing capacity bids). Then a second optimization program, representing a full co-optimization process, is run considering a simplified version of the unilateral links and the deterministic requirement to clear both day-ahead and balancing capacity markets while allocating the available cross-zonal capacity. The simplified version of the unilateral links is computed based on the output of the first optimization program. - Scenario B2. Two-step unilateral linking: This scenario represents a variant of the previous design. First, the balancing capacity market is cleared and the cross-zonal capacity is allocated while accounting for energy and the deterministic requirement for flow-based compatibility. However, the current EUPHEMIA setup is used as a third optimization program (a second step) to clear the day-ahead market while accounting for the results of the two previous optimization programs.
Modelling of Co-optimization Welfare maximization or equivalently bid-cost minimization has been considered in the three scenarios regarding the procurement of balancing capacity. Moreover, since the level of activation of the TSO demand in real time is not known in advance, it is important to ensure that, for any activations of TSO demands lower than the TSO demands that are matched in balancing capacity market clearing, the network can support the resulting flows that are required for balancing this configuration of TSO demands.

Therefore, on the one hand, co-optimization optimizes the allocation of limited generation resources and limited CZC to energy and balancing capacity, and thus de facto improves welfare relative to a suboptimal sequential allocation. On the other hand, the “deterministic requirement” - i.e., the requirement that any TSO demand matched in the day-ahead balancing capacity market should be such that if it is activated fully or partially in real-time the network should be able to support all the possible resulting flow patterns- is a way to cope with the allocation of CZC for balancing capacity for which actual cross-zonal flows are uncertain at the time of CZC allocation. The deterministic flow-based compatibility requirement has been enforced in all of the three studied scenarios. This requirement has the formidable computational challenges and therefore, N-SIDE proposes a method for decisively overcoming this challenge through the so-called “inscribed boxes approach”. The idea of this approach is to propose an approximation of the deterministic requirement which ensures that the deterministic requirement is satisfied (thus it tightens the constraint) while preserving a computationally tractable model with clear economic interpretations.

Welfare optimization

In the Single Day-ahead Market Coupling (SDAC), the criterion underlying market operations is the maximization of the welfare. In a context of inelastic demand, as envisioned for the procurement of balancing capacity, and in a pay-as-bid scheme, welfare maximization is equivalent to procurement cost minimization: this design choice is called bid-cost minimization. Due to challenges associated to another design options called (marginal-price) “procurement cost minimization” (discussed in the N-SIDE final report), and in view of the constrained timeline for the co-optimization prototypes study, the welfare maximization criterion has been retained for the co-optimization prototypes and simulations.

MTU resolution

The prototype study was conducted on the basis of one market time unit of 1 hour, only one balancing capacity product in one single direction in combination with flow-based. It has not been investigated how 15min products are optimized and a setting with multiple balancing capacity products in which also substitution of reserves would be performed.

Prototype

Based on the scenarios described above, three prototypes were created and used to perform simulations in order to assess the technical feasibility and performances of the co-optimization of energy and balancing capacity considering cross zonal available capacity for both usages.

The Euphemia prototype is based on Euphemia 11.1. Euphemia has been modified essentially along two dimensions:

- Network models have been adapted so as to create for each bidding zone of the input topology, a bidding zone for the energy market, and a bidding zone for the balancing capacity market (mFRR up). Moreover, constraints related to the network model (Flow based compatible) have been implemented to enforce the deterministic requirement.
- Multilaterally linked orders have been introduced as new bidding products. This prototype has been used as the basis for all simulations.

For Scenario B2 (2-step with unilateral bid linking), the first step is performed with a simplified version of this prototype, where all bid indivisibility requirements are dropped so that a large continuous problem is solved when taking this simplification into account.

SCOPE

Use case considered for the co-optimization study
Datasets: historical network data of the day-ahead market - Initial cross zonal capacity (CZC) and network coupling related data - Supply bids and TSO capacity demand for the Balancing capacity market - Supply and demand bids for the Day-ahead market - Essence of the link between concerned linked supply balancing capacity and energy bids
Order books - Group the historical DAM order books and the new energy/reserve bids (the order books are not realistic at this stage but enable to support the quantitative assessment of performances and to analyze specific features of the designs.)
Design options envisioned by ENTSO-E - Scenario A1: 1-step multilateral linking approach - Scenario A2: 1-step unilateral linking approach - Scenario B2: 2-step unilateral linking approach
Key Performance Indicators - Time To First Solution (TTFS) which is a key performance indicator in SDAC on scalability, as it gives the minimum time to find a first feasible solution and avoid decoupling. - CZC allocations and zonal price differences

OUTCOMES AND CONCLUSIONS

Observations on algorithm performance

The prototype shows that co-optimization is tractable for all SDAC in a 60 min context and the deterministic requirement. However, this includes certain rather limiting design choices.

The initial performance assessment indicates that the key performance indicator for the algorithm performance in TTFS (Time to First Solution) between Euphemia and Euphemia with co-optimization is approximately 3.5 times longer in 1-step multilateral linking of DA and balancing market compared to DA market as seen in the illustration on page 60 in Co-optimization_roadmap_study_final_report. The studied 1-step unilateral linking is approximately on equal performance level than 1-step multilateral linking, whereas 2-step unilateral linking is approximately 4 times longer in TTFS compared to DA market as seen from the illustration on page 62 in Co-optimization_roadmap_study_final_report.

As for other results of the study, the average absolute flows show that CZC is allocated both to DA and balancing capacity in all three scenarios. As expected, the allocation is not coherent in terms of zonal price spreads in the 2-step unilateral scenario.

Moreover, the simulation was done with 60min MTU only. It is already proven that implementation of 15 min MTU means significant challenge for the performance of Euphemia and will require extension of the computation time limit. Implementing co-optimization on top of 15 min MTU will require additional extension of the time limit. If the ratio mentioned above shall be applied (3.5 times as mentioned in the first paragraph of this section), it would bring DA auction to a very long calculation time (possibly more than 2 hours).

No further assessment of inner product linking has been conducted which however in practice shall be required to establish effective balancing capacity markets and adds more complexity to the algorithm in addition to the current incomplete requirements captured by the prototype study. It is expected that the correction of requirements and the addition of more requirements will lead to higher computation times and impact of the SDAC processes and have to be further investigated.

Short conclusion on the outcomes of the study

The study produced a Euphemia Prototype for Co-optimization, taking into account the flow-based compatibility deterministic requirement, which performs well with 60’ MTU data and one additional BC product besides the DA. The simulations validate the proof-of-concept implementations of the scenarios in scope. Furthermore, they also show that the 1-step scenarios, compared to the 2-step scenario, avoid incoherent cross-zonal capacity allocations with respect to zonal price spreads, risks of infeasible second steps, and are also faster overall. On high-level, the scenarios can be simulated. This initial simulation still lacks key elements like the 15min MTU, multiple balancing market capacity products etc., and in general it is a simplification of the real market. Thus, prototype study was only a first preliminary exploration, also in the way it was designed, so the results cannot be used for any decision-making process before the simulation data and key assumptions brought up to date. There are still many design questions need to be addressed (see the next section).

CONCERNS RAISED BY SDAC MSD

As a general observation, it should be noted that the study at hand has several significant limitations. First, the assumptions and model specifications must be brought into perspective, as the prototype study applied some far-reaching simplifications. This includes for example the replacement of a procurement cost minimization approach by a welfare maximization approach (for balancing capacity market). As a consequence, it is obvious that not all requirements from the Commission Regulation 2017/2195 establishing a guideline on electricity balancing (EB GL) have been included in this prototype study, or the requirements have not been modelled in fully correct manner. The prototype is only capable to form prices based on welfare maximization which is not compliant to legislation. Therefore, the results of the prototype study cannot be considered as a starting point of any implementation efforts, since the study does not offer sufficient ground for drawing any far-reaching conclusions. Any actual implementation steps, specifically any amendments to existing regulation and methodologies strictly require further analysis and a complete technical assessment.

When deploying the entire topology, functionalities of only a single balancing product were analyzed out of total of 6 products. The optimization of bid linking across the balancing capacity products was not considered which will put more stress to the algorithm performance and is likely to seriously delay the market coupling calculation process. Because of this, required EB Regulation and CEP requirements such as substitution of reserves could not be assessed.

The following paragraph offers a more detailed list of concerns on the side of TSOs and NEMOs. There were several items studied which still needs further clarification:

- Unilateral bid linking option, this seems to come with down-sides and complexity. In the unilateral bid linking option, the balancing market is prioritised over the DAM.
- The basic principles of multilateral bid linking, and its complexity needs to be further investigated.
- The prototype only considers balancing capacity for mFRR up. The scalability of the co-optimization for more than one balancing capacity product needs to be further evaluated.
- Use of different balancing capacities in opposite ends of the bidding zone border needs to be defined.

- Interaction between co-optimization and market-based CZC allocation method (EBGL 41(1)), where the procurement of balancing capacity happens based on actual balancing capacity bids and forecasted DA energy bids (or, vice versa for the inverted market-based CZC allocation method).

- To be clarified: TSOs' requirement of deterministic compatibility with flow-based ensures that the network must cope with any configuration/uncertainty of TSO's balancing activations in real time. Even if the used method of "inscribed boxes" performs well in the assessed criteria (tractability, hub dependence, uniqueness of balancing capacity prices), the approach could be too conservative to allocate capacity for BC with negative effect on the efficiency of day-ahead energy market.

- General risk of conflict and incompatibility between co-optimization requirements and current regulatory requirements.
- The simulations were done using 60 min MTU historical data. There are not any performance results reflecting the status with 15 min MTU data which would be essential as any potential implementation of co-optimization would need to be done in a 15 min MTU framework.

- Impact of co-optimization requirements (i.e. linking, one/multi steps, deterministic/probabilistic low-based compatibility) on market results were outside the scope of technical assessment study. While apart from technical feasibility, each market design option can significantly change the outcomes of the markets (welfare, …).

- SDAC's current performance in terms of security, reliability and quality must always remain as top priority, when assessing implementation of new requirements as the co-optimization. The prototype study did not investigate any of these aspects.

- In terms of prioritization of the workload of experts involved in SDAC & SIDC and current challenges of other projects planned to be implemented before co-optimization (e.g IDA implementation, 15 min MTU implementation, Nordic FB, …). It is not certain that implementation of co-optimization can be done in parallel to the other ongoing projects without putting them at risk.

N-Side suggested that many design questions should still be addressed before reaching a full set of requirement specifications. The following list includes a range of possibilities for next steps from the design perspective:

High level Design:

- Improved bid linking (advanced linking, unilateral bid linking design with multiple products, …)
- Incoherent prices due to sequential calculations
- The difference between optimization of the Bid-cost minimization and Welfare maximization needs to be studied. Additional study item is Bid-cost minimization versus Marginal-price-cost minimization.
- Impact of flow netting (to identify conditions in which flow netting should be favored or deactivated)

Detailed Design:

- Curtailment management
- Rounding procedures
- Ramping

Public consultation is ongoing for co-optimization, market organisations (EURELECTRIC/EFET) have indicated severe issues of simultaneous markets which are a result of a single Gate Closure Time of all involved markets, the basis of the co-optimised allocation process. Market organization strongly advice to not continue the technical implementation based on the proposed requirements and warn of very inefficient bidding with the current requirements. More analysis needs to be done to see what exactly market will need in the future and if more advanced cross-product types will be requested.

In conclusion, a general concern from MSD is how the integration of the co-optimization functionality can be ensured in the algorithm looking to the challenges SDAC faces already with current requirements and the requirements which are expected to come based on the roadmap. This concern applies to both algorithm performance and the workload of the SDAC MSD. Therefore, the co-optimization is not on the radar yet. Prioritization of the regulatory development tasks shall be discussed and agreed with ACER as needed.

Current activities and the next steps

SDAC MSD, together with ENTSO-E has now concluded the assessment of the results from the co-optimization study. The risks, concerns and open points have been described above. The next steps include the following tasks and items:

- Communication with NRAs and ACER about the results and concerns SDAC has identified
- Planning of the next version of the simulation targets
  - Closing the open questions
  - Defining the feasible timeline, resources, and cost estimation
- All TSOs to submit the proposal of HCZCAM Article 38(3) of the EB GL to ACER by 17th of December 2022.

Co-Optimization of Energy and Balancing Capacity in the European Single Day-Ahead Coupling

Roadmap study

Wednesday May 18, 2022

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Abstract

The present study on co-optimization of energy and balancing capacity in SDAC achieves three objectives.

Firstly, the study enhances Euphemia by successfully introducing co-optimization of energy and balancing capacity in the day-ahead algorithm, and does so by satisfying an extremely challenging requirement related to the ability of the network to support the exchange of real-time balancing energy, referred to as the deterministic requirement. The prototype that has been developed relies on state-of-the-art optimization innovations and tackles the co-optimization of energy and one balancing capacity product (upward mFRR) at the SDAC level. This prototype is used to provide a first quantitative scalability assessment of co-optimization.

Secondly, the study seeks to clarify market design concepts considered by stakeholders and to provide answers to fundamental questions on bid linking options, enforcement of the so-called deterministic market price implications of the flow-based compatibility requirement, 1-step versus sequential market operations, welfare maximization in comparison to (marginal-price) procurement cost minimization for the procurement of balancing capacity, flow netting, or risks of price reversals in balancing capacity markets. Given the breadth of the topics covered, open questions or challenges are pointed out when the study had to limit the efforts on providing preliminary answers on a specific topic. Nonetheless, most of the concepts are extensively discussed, and the study also describes the success in identifying an approach to efficiently enforce the deterministic flow-based compatibility requirement, that has been validated by numerical experiments relying on the prototyping work described above.

Thirdly, the study proposes a roadmap together with timeline and cost estimates in the last part of this report, gathering learnings from the first and second parts.

An executive summary of this study has been provided separately in the form of slides.

Context

Article 40 of the EBGL [1] considers the introduction of energy and reserve co-optimization in the European day-ahead electricity market. In 2020, ENTSO-E launched two studies in relation to this aspect: the linking of bids in a co-optimization setup [2], and flow-based compatibility [3].

In summary, the linking of bids study [2] proposes several procedures for clearing energy and reserves. Apart from a proper co-optimization procedure, the idea of the alternatives that are examined is to separate the clearing of balancing capacity from the clearing of energy. This sequential clearing implies that either energy or balancing capacity is awarded first, and the report [2] develops a discussion about how bids can be linked in either a *multilateral* sense (in the spirit of a co-optimization) or a *unilateral* sense (in the spirit of prioritization through products).

Furthermore, the flow-based compatibility study [3] proposes a market clearing model for representing the so-called "**deterministic requirement**", i.e., the requirement that any TSO demand matched in the day-ahead balancing capacity market should be such that if it is activated fully or partially in real-time the network should be able to support all the possible resulting flow patterns. We will refer to this in the sequel as the *deterministic requirement*. The study [3] identifies this requirement mathematically as a robust optimization problem and highlights the formidable computational challenges that it introduces. In a follow-up analysis [5], N-SIDE proposes a method for decisively overcoming this challenge through the so-called "**inscribed boxes approach**". The idea of this approach is to propose an approximation of the deterministic requirement which ensures that the deterministic requirement is satisfied (thus it tightens the constraint) while preserving a computationally tractable model with clear economic interpretations.

The implementation impact assessment [4], which was based on the analysis presented in both the linking of bids and the flow-based compatibility studies, narrows down a set of options for the implementation of Article 40 of the EBGL [1]. As the result of a modification of the original requirements by ENTSO-E along with a gathering of a deeper expertise on the subject, the original set of options proposed in the implementation impact assessment [4] evolved to produce the three following scenarios combining both requirements (the linking of bids and the deterministic requirement) that will be studied in this report:

- **Scenario A1 → *One-step multilateral linking***: Full co-optimization in a single optimization process with multilateral linking of bids in which the deterministic requirement is taken into account through the inscribed boxes method.
- **Scenario A2 → *One-step unilateral linking***: A sequential optimization process with first an attempt clearing of the balancing capacity market (without taking the day-ahead market into account to enforce prioritization of linked balancing capacity bids). Then a

second optimization program, representing a full co-optimization process, is run considering a simplified version of the unilateral links and the deterministic requirement to clear both day-ahead and balancing capacity markets while allocating the available cross-zonal capacity. The simplified version of the unilateral links is computed based on the output of the first optimization program.

- **Scenario B2 → *Two-step unilateral linking***: This scenario represents a variant of the previous design. First, the balancing capacity market is cleared and the cross-zonal capacity is allocated while accounting for energy and the deterministic requirement for flow-based compatibility as presented above. However, the current EUPHEMIA setup is used as a third optimization program (a second step) to clear the day-ahead market while accounting for the results of the two previous optimization programs.

The goals of the first part of the present document are to compare the different alternatives in terms of technical feasibility, and to better understand what they imply in terms of economic efficiency of the day-ahead auction, consistency of economic signals, congestion revenues, along with other important economic performance metrics. The output of this first part creates the basis for a more careful study of the technical feasibility of each option, which is assessed through prototyping an extension of EUPHEMIA and performing simulations on realistic-scale datasets. Then, the second part of the current report details the prototyping efforts and analyze the simulation results. To conclude, based on the insights of Part I and Part II, we provide in Part III of the present report an informed timeline as well as an estimate of the costs for the required efforts to elaborate a complete market design for the co-optimization of energy and balancing capacity, and for its implementation in the SDAC algorithm.

1. What to Expect from Co-optimization

Co-optimization amounts to a multi-product auction. Multi-product auctions are abundant in the practice of economics, and the contributions of Nobel prize laureates Professors Paul Milgrom and Robert Wilson (Stanford University) to the topic [6] include applications in spectrum auctioning as well as electricity markets. Multi-product auctions are useful for allocating goods efficiently in the case of substitutability, complementarity, and other economically relevant features. For instance, in charity auctions bidders are able to bid on (potentially) mutually exclusive prizes, while in spectrum auctions the combinations of certain bandwidths deliver superior economic value to their owners than the sum of the values that these bandwidths can furnish independently. The multi-product auctioning of energy and balancing capacity is an auction along the same lines [6]. In the case of electricity markets, essential economic features include, among others, mutual exclusivity, substitutability, and dependency. Mutual exclusivity refers to the fact that, since the generation capacity of a power plant is finite, one MWh allocated to the production of energy is one MW that cannot be made available for balancing capacity. Furthermore, regarding substitutability, it represents the characteristic that a fast generation technology can contribute identically to the balancing capacity requirements of manual balancing capacity products such as mFRR but also automatic balancing capacity products such as aFRR. Finally, dependency is also essential since it depicts the concept that, contrary to mutual exclusivity, offering for example downward balancing capacity presupposes that the power plant is generating at the first place to be able to reduce its production.

In this section, illustrative examples are used to emphasize some important points related to economic efficiency and coherence of clearing prices in a co-optimization of energy and reserve capacity context. Since only single-period examples are employed to highlight these elements, MW and MWh are used interchangeably in this section and a MWh should therefore be understood as a unit of measuring balancing capacity equivalent to one MW of balancing capacity made available for one hour. We would also like to clarify that €/MWh is equivalent to (€/MW)/h. Consequently, we solely use "€/MWh" as unit representing either the unitary cost of delivered energy (DAE) or the unitary cost of reserved balancing capacity over a given period.

1.1 Efficient Allocation

One of the major advantages of co-optimization is the fact that it can deliver maximum economic efficiency because it ensures that all interactions between energy and balancing capacity are accounted for when these interdependent resources are allocated in a multi-unit auction. The grey box in section 9.2.1 of the implementation impact assessment [4] provides an example of why unilateral linking (in the sense of the mathematical definition provided in section 2.2 of the present report) can result in a deterioration of efficiency relative to multilateral linking (in the sense of the mathematical definition provided in section 2.1 of the present report). We now revisit this example.

Example 1.1: Unilateral bid linking deteriorates efficiency

Let us consider a market with the products indicated in Table 1. The merit orders of the energy and balancing capacity markets are indicated in Figure 1.

Asset A (5 MW)	20 €/MW for BC	25 €/MWh for DA
Asset B (5 MW)	22 €/MW for BC	40 €/MWh for DA
Demand	5 MW BC at any price	5 MW DA at any price

Table 1: An example that illustrates that unilateral linking deteriorates efficiency compared to multilateral linking.

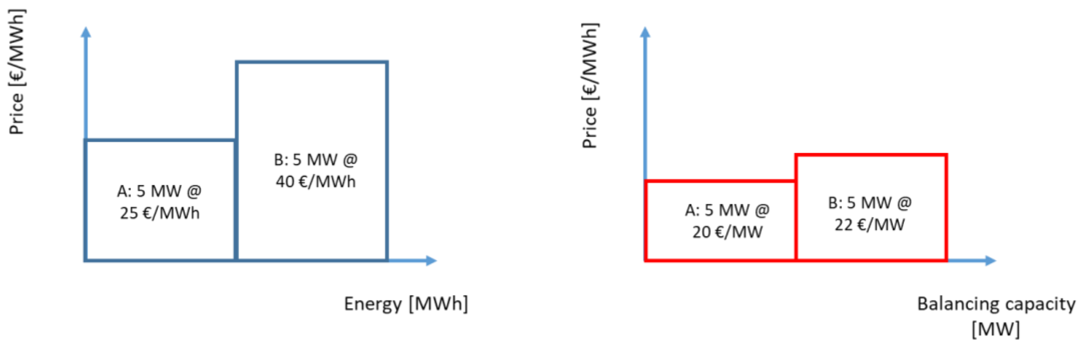

Figure 1: Linked merit orders of the energy (left) and balancing capacity (right) market for Example 1.1.

Suppose that the link used to represent Asset A and Asset B's behavior is unilateral linking. In that situation, one optimizes BC first, without regards to the energy bids. In such a case, Asset A is selected, because it stands before Asset B in the merit order insofar as BC only is concerned, as indicated in the left panel of Figure 2. With such a sequential optimization, Asset A is no longer available for DA, as indicated in Figure 2. Therefore, Asset B is selected for DA at a price of 40 €/MWh. The total sourcing cost, in this case, is equal to 300 € (20x5=100 € for BC and 40x5=200 € for energy).

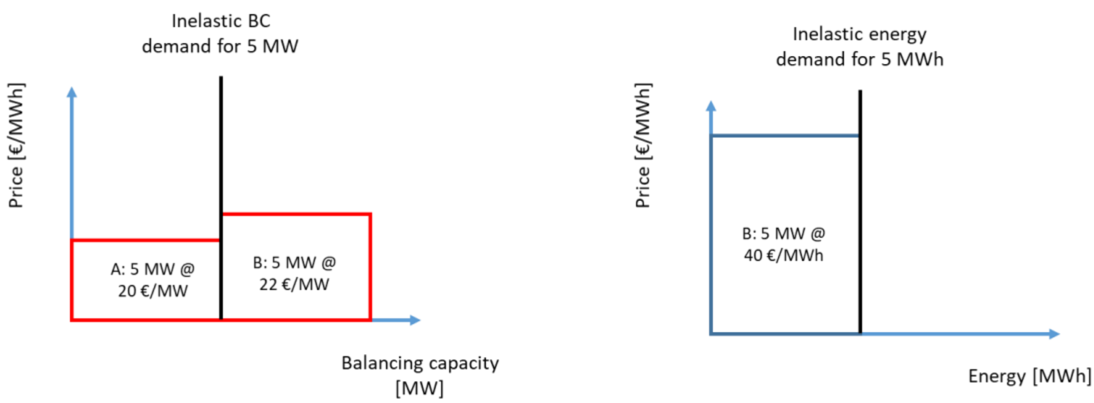

Figure 2: Suboptimal allocation resulting from unilateral bid linking.

On the other hand, considering multilateral linking for each asset (which uses a unique objective function to accept BC and energy bids) will result in a different selection, and allocate Asset B to the BC market and Asset A to the DA market, for a total cost of 235 € (22x5=110 € for BC and 25x5 = 125 € for DA). This is because the merit order of BC and of energy can no longer be explored in isolation when BC and energy bids are linked. This erroneous consideration of merit orders in isolation is an inherent attribute of unilateral bid linking since it implies a prioritization of the balancing capacity market. Note that the performance of unilateral linking in this case can be made arbitrarily suboptimal by increasing the energy bid price of Asset B from 40 €/MWh to an arbitrarily large value.

The linking of bids report [2] and the IIA [4] argue that unilateral linking represented by a sequential optimization could incorporate elements of both markets at each step. This would amount to duplicating computationally hard operations, and still introducing possible approximation errors, and this should obviously not be preferred if multilateral linking can be shown to be computationally efficient while satisfying the business requirements of the auction.

1.2 Coherent Prices

Since multi-product auctions essentially allow sellers and buyers to access multiple markets simultaneously, prices of these products need to be consistent in order to ensure that the auction allocation is consistent with the selfish private objectives of market participants. This notion is familiar in the implicit allocation of transmission capacity, as year of experience in EU market design have proven: the price of access rights to network infrastructure needs to be in equilibrium with the zonal price of electricity. For instance, the only way that price differentials can be maintained in a two-zone model is if the link is congested, otherwise there would be an arbitrage opportunity to buy power at the cheap market and sell it at the expensive one. This is a no-arbitrage condition that connects the price of electricity in different zones, and the economic value of transmission capacity. An analogous no-arbitrage condition connects the price of energy and the price of balancing capacity in a co-optimization model. This condition states that the price of BC and the price of energy should be such that the profit margin of any market participant who is matched in both auctions should be equal, otherwise that market participant would simply opt for the market (be it energy or balancing capacity) where the prices are more favorable.

Such no-arbitrage conditions are encoded in the optimality conditions of the first scenario (Scenario A1)[1]. They are an inherent property of a co-optimization model. The solution of a co-optimization model satisfies these properties by default. Instead, when we break an integrated optimization problem into two sequential pieces, we not only put the guarantee to obtain optimal solutions at risk, but also endanger the no-arbitrage conditions which ensure coherent prices (i.e. "volume coupling effects"). We highlight this problem with an illustrative example.

[1] Scenarios in scope in the present report are discussed in detail in Section 4.

Example 1.2: Unilateral bid linking creates inconsistent prices

We revisit Example 1.1 by modifying it slightly to lift price indeterminacies. We consider the bids of Table 2.

Asset A (5 MW)	20 €/MW for BC	25 €/MWh for DA
Asset B (5 MW)	22 €/MW for BC	40 €/MWh for DA
Demand	4 MW BC at any price	4 MW DA at any price

Table 2: An example that illustrates that sequential optimization produces incoherent prices.

The allocation under unilateral linking performed by Scenario A2 (One-Step Unilateral Linking) is presented in Figure 3. The prices resulting from this sequential optimization are 20 €/MWh for the BC market, and 40 €/MWh for energy. These prices are not coherent. Let us examine why:

- Asset A is asked to split its capacity between the BC market (4 MW) and the energy market (1 MWh). But the profit margin in the energy market is clearly preferable for Asset A (40-25=15 €/MWh in the energy market, versus 0 €/MWh in the BC market). It is thus not sustainable to ask Asset A to split its capacity according to the result of the auction (which is suboptimal, as we argue using the reasoning of Example 1.1). Consequently, one may expect that the bidding strategy of Asset A will incorporate such an arbitrary opportunity, so as to maximize its total profit (hence in practice increase the BC bid price).
- Asset B is asked to stay out of the BC market, and supply 3 MWh in the energy market. The price signals are coherent with the incentives of Asset B, since the profit margin for Asset B in the BC market is negative (20-22=-2 €/MWh) while the profit margin is 0 €/MWh in the DA market.

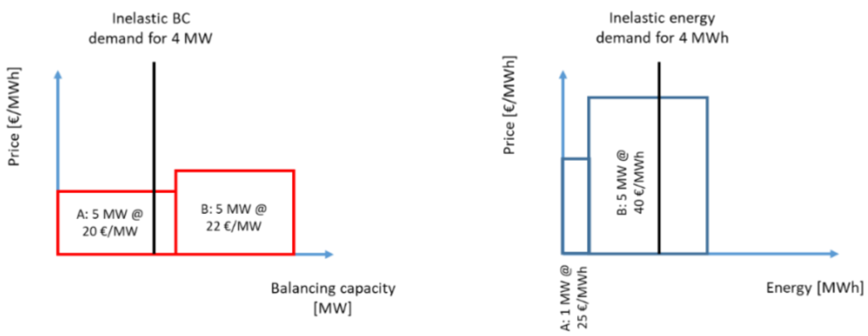

Figure 3: Unilateral linking produces incoherent prices. If the BC market is optimized first, the clearing price is 20 €/MWh. If the DA market is optimized next, the clearing price is 40 €/MWh. These prices are not compatible with the matching results: asset A is not making equal profit margins in the two markets, even though it is splitting its capacity in both markets.

What would be expect in equilibrium? If Asset A starts migrating its capacity into the energy market in order to capture the favorable profit margins, the BC market price will eventually be set by Asset B at 22 €/MW. And if Asset A is to remain in the energy market, it will only be willing to do so if its profit margin is worth it (i.e., no lower than 2 €/MWh, which is its potential margin in the BC market if asset B sets the price there). This means that the energy price cannot go lower than 27 €/MWh (its energy cost plus a minimum profit margin of 2 €/MWh). But since Asset A can also make a profit margin in the BC market (it is expected to supply its capacity there too, since it is cheaper than its competitor), the DA price cannot go above 27 €/MWh either. Indeed, the co-optimization model using multilateral linking produces the following allocations and market prices: the DA demand is supplied by Asset A, the BC demand is supplied by Asset A (1 MW) and Asset B (3 MW), the DA price is 27 €/MWh and the BC price is 22 €/MWh. Let us analyze why these prices are coherent:

- Asset A is earning a profit margin of 2 €/MWh in both the DA market (27-25=2 €/MWh) and the BC market (22-20=2 €/MWh). At the same time, its entire capacity of 5 MW is used up and earns a profit, which is what Asset A wants, since both the DA market and BC market are profitable for this asset.
- Asset B is not willing to engage in the energy market since it would be making a loss there (27-40=-13 €/MWh). On the other hand, it is indifferent about how much capacity it offers in the BC market, since it earns a zero profit in the BC market (22-22=0 €/MWh). This allows the auctioneer to ask for a BC quantity from Asset B which is strictly positive but less than its full capacity.

A dangerous symptom of incoherent pricing that results from unilateral linking are so-called **price reversals** in balancing capacity markets. Price reversals refer to situations where lower-quality / slower-moving balancing capacity (such as mFRR) receive a higher clearing price than higher-quality / faster-moving balancing capacity (such as aFRR). This creates the obvious perverse incentive of inducing assets to withdraw capacity from higher-quality reserve markets, which is highly undesirable in terms of system security. Price reversals were observed and gamed in the original sequential market design of the California ISO in the early 2000s, and were corrected by the implementation of co-optimization in CAISO [7]. We provide an illustrative example of price reversals in Appendix A.

1.3 Simplified Bidding and Internalization of Opportunity Costs

Section 1.2 explains why unilateral linking produces incoherent prices, in the sense that the auction allocation is not aligned with the prices that agents are rewarded in each of the auction. One could argue that this inconsistency (and also the suboptimal allocation pointed out in Section 1.1) could be corrected if agents would internalize the opportunity cost of allocating capacity to the BC market. In a very simple setting, this might work. In the real world, it is intractable because of the multitude of BC products that are already traded in EU markets, which makes perfect anticipation of opportunity costs practically impossible. This introduces risk and complexity for traders, and even small errors in anticipated energy clearing prices can result in highly suboptimal outcomes.

Co-optimization takes care of internalizing opportunity costs optimally, without placing the burden on traders to anticipate these opportunity costs. This relieves traders from risk as well as the burden of developing complex bidding strategies: the only thing that agents are required to do is bid the physical costs and constraints of their assets into the co-optimization auction, and the rest is taken care of internally by the auction algorithm. Indeed, as proven by economic theory, marginal cost-based bidding is the optimal strategy for a fringe asset under uniform pricing auctions as long as economic and technical constraints are sufficiently accurately expressed.

Example 1.3: Unilateral bid linking makes bidding complicated and introduces risk

We revisit the market of Example 1.2. With unilateral bid linking, agents' energy bids can be affected by the outcome of the BC market, but not the other way around. Therefore, the agents might wish to internalize the opportunity cost that they anticipate in the energy auction when submitting their BC market bids, in order to make up for the missing direction in unilateral bid linking. We represent the anticipation of market agents in Table 3, where it is assumed that both agents anticipate perfectly the equilibrium price of co-optimization (which induces the economically efficient allocation of resources in the market).

Asset A (5 MW)	20 €/MW for BC	25 €/MWh for energy	Anticipated energy price: 27 €/MWh	Adjusted BC bid: 22 €/MW for BC
Asset B (5 MW)	22 €/MW for BC	40 €/MWh for energy	Anticipated energy price: 27 €/MWh	Adjusted BC bid: 22 €/MW for BC
Demand	4 MW BC at any price	4 MW energy at any price		

Table 3: An example that illustrates that unilateral bid linking creates complicated bidding and introduces risk for market agents. Both Assets A and B correctly anticipate the equilibrium DA price, and adapt their BC market offers accordingly.

Running the sequential optimization of Scenario A2 (Unilateral linking One-Step) with the bids of Table 3 can indeed result in an efficient matching[2]. However, this assumes an idealized situation where all assets correctly anticipate the equilibrium energy price. In a market with a single period and two products (energy and a single type of reserve) this may be plausible, but in a market that spans 96 periods, dozens of geographical regions, and up to eight balancing capacity products (upward / downward and aFRR / mFRR / RR) per time period, such a perfect anticipation assumption is wishful thinking. One-step multilateral co-optimization (i.e., Scenario A1) relieves the bidders from this significant level of complexity.

Unilateral bid linking also introduces risk. Let us consider what happens, for instance, if Asset A anticipates a slightly lower energy price than the equilibrium energy price of 27 €/MWh. The situation is presented in Table 4. Here we assume that Asset A makes an error of 0.1

[2] Note that the matching is economically efficient, in the sense of maximizing economic welfare, but it is interestingly not coherent in terms of pricing. Using the bids of Table 4 produces a BC price of 22 €/MWh, but a DA price of 25 €/MWh (not 27 €/MWh, which is the equilibrium price). Thus, even though internalization of opportunity costs in the BC market can induce an efficient allocation, it would be also necessary to internalize opportunity costs in the DA market in order to induce coherent equilibrium prices.

€/MWh in its anticipation of the DA price, and instead of correctly anticipating it at 27 €/MWh, it anticipates it at 26.9 €/MWh. This completely overturns the allocation, and the BC demand is matched with Asset A, which is inefficient, instead of keeping the capacity of Asset A aside for covering demand in the DA market. This level of risk poses a serious threat to economic efficiency and the ability of market participants to communicate the necessary information to the market so that their assets may be scheduled efficiently.

Asset A (5 MW)	20 €/MW for BC	25 €/MWh for DAE	Anticipated DAE price: 26.9 €/MWh	Adjusted BC bid: 21.9 €/MW for BC
Asset B (5 MW)	22 €/MW for BC	40 €/MWh for DAE	Anticipated DAE price: 27 €/MWh	Adjusted BC bid: 22 €/MW for BC
Demand	4 MW BC at any price	4 MW DAE at any price		

Table 4: An example that illustrates that unilateral bid linking creates complicated bidding and introduces risk for market agents. Assets A makes a slight error in its anticipation of the equilibrium energy price (indicated in red) and provides an offer that moves the market clearing outcomes under unilateral bid linking to its originally inefficient level before an attempt was made to internalize opportunity costs.

1.4 A Universal Welfare Objective Which Can Prioritize TSO BC Demand

One-sided markets typically involve a fixed quantity of a single product that is being bought, with sellers providing competing economic offers. The auction then uses the merit order in order to supply the requisite demand at minimum cost. The generalization of this setting to two-sided markets, where both buyers and sellers provide economic offers, is straightforward, and the merit order intersected by a vertical demand is now replaced by a supply and a demand-side merit order, where the market clearing prices is determined by the point in which these merit order curves intersect.

Co-optimization generalizes this welfare maximization to multi-product auctions. Here, CZC is allocated in a way that the marginal value functions of CZC in energy and BC markets intersect, see exhibit 3.2 of [3] for a graphical illustration. What is important to explicitly point out is that there is nothing contradictory about multi-product two-sided auctions: they are perfectly able to generalize welfare maximization from a setting of one product (energy only) to multiple products (energy *and* several balancing capacity). In fact, we already perform such a multi-product auctioning of energy and transmission capacity in the pan-European day-ahead energy market. Although transmission capacity is not auctioned off explicitly, it *is* auctioned off implicitly, and allocated in a way that welfare is maximized in a two-sided energy auction. Market coupling is indeed an implicit CZC allocation auction, where the CZC is offered “at any price” and where the CZC demand is implicitly embedded within the energy bids.

In fact, it is possible to show that the co-optimization model that EUPHEMIA solves in the day-ahead market is equivalent to the explicit auctioning of energy and transmission capacity rights separately, assuming perfect anticipation of opportunity costs, see pages 82 and 83 of

[10], also slides 25-31 of [11]. The reason that these processes have been combined throughout the years (market coupling has been progressively replacing explicit CZC allocation auctions in Europe) is that it is difficult for traders to anticipate the efficient level of opportunity costs, especially since our auctions span multiple inter-dependent time intervals and geographical areas. Strong empirical evidence of this challenge is available in recent studies about the inefficiencies that were caused by the forced exit of the UK from SDAC, and where various sequential optimizations (i.e., "volume coupling" variants vs. explicit auctions) are being analyzed[3].

There is nothing different in the principle of introducing balancing capacity as a product to the day-ahead auction. The motivation is also the same: since energy and balancing capacity are interdependent (especially in a regime of large-scale renewable energy integration, where value migrates from energy to reserve services), the separate auctioning of energy and balancing capacity introduces severe opportunity cost anticipation challenges, and thus inefficiencies in resource allocation.

Having argued about the generality of the co-optimization framework, we now proceed to underscore that **the ability of the TSOs to prioritize their demand for balancing capacity is not undermined in this framework**. If anything, it benefits from the more efficient allocation of resources. The fact that we replace a sequential auctioning of TSO demand first with a simultaneous auctioning does not mean that TSO demand for balancing capacity is undermined. We illustrate this point with the following example, which is inspired from [3].

Example 1.4: Co-optimization does not undermine the ability of TSOs to secure balancing capacity

Consider the system that is depicted in Figure 4. In this system, we have a TSO demand for BC of 5 MW (QR TSO: 5 MW), a generator with a marginal cost of 100 €/MWh (G1: 100 €/MWh) and a load of 100 MW (L: 100 MW) in the right zone; and a generator with a marginal cost of 20 €/MWh (G2: 20 €/MWh) which is also offering 10 MW of balancing capacity (QR BSP2: 100 MW) in the left zone. The challenge is to ensure that the TSO demand for BC is satisfied, since the CZC (100 MW) might be instead allocated for the transfer of energy from cheap energy producer G2 to the load in the right zone.

<u>**Co-optimization ensures that this will not happen, as long as the valuation of the TSO for BC is higher than the valuation of energy buyer L for energy**</u>. Concretely, the TSO can submit a "price-inelastic" demand for BC, i.e., a demand for BC that is valued at the highest possible price that the auction allows. If it does so, the algorithm will treat this demand as a hard requirement, which will only be left unsatisfied in case it is needed for the auction model to remain feasible (e.g., in the case of inadequate BSP supply).

[3] See for example the Cost-Benefit Analysis of the Multi-Regional Loose Volume Coupling.

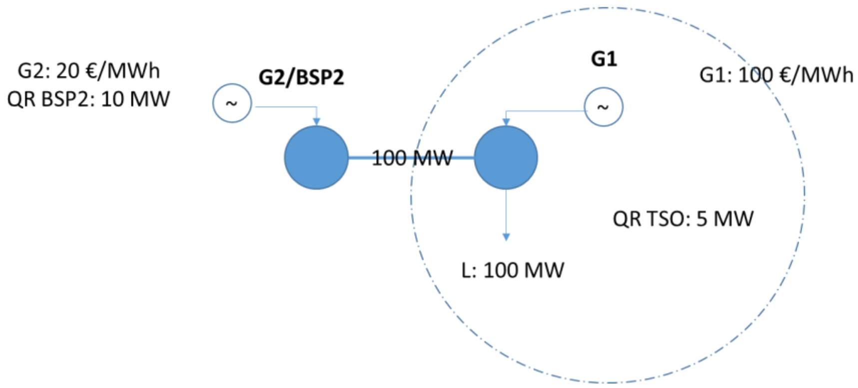

Figure 4: Example that illustrates that TSO demand for BC is not compromised in co-optimization.

More specifically, in this example the co-optimization correctly allocates 5 MW of BC of BSP2 to cover the TSO demand in the right zone. The price for energy is in the right zone 100 €/MWh, which is required since G1 is at the money in the energy auction, with 5 MWh of G1 supplied to cover the demand of L that cannot be fully satisfied by G2. The price of BC in the left zone is 20 €/MWh, while the price of energy in the left zone is 0 €/MWh. These prices ensure that the multilaterally linked offer of the left zone (energy bid G2, which is linked to BC bid BSP2) is voluntarily willing to split its capacity between energy and BC, while being at the money in the energy market.

To conclude, co-optimization is capable of supporting a prioritization of balancing capacity. In doing so, it simplifies bidding and avoids requiring from agents to internalize opportunity costs. CZC is allocated in the process in a way that is consistent with the valuations of agents for the various products that are traded simultaneously in the market.

1.5 Bid cost minimization vs. payment cost minimization

Another fundamental linked aspect relates to the actual objective being pursued when TSOs are procuring BC. On the one hand, one can argue that welfare maximization is an essential "first principle objective" that – by definition – creates the largest possible market value. In single-buyer markets with inelastic demand (i.e., BC markets), welfare maximization is fully equivalent to **"bid cost minimization"**. In other words, the welfare maximal solution is also the solution that selects the cheapest possible bids (and is thus also the optimal "paid-as-bid" solution).
On the other hand, however, one can also argue that what matters is the actual procurement cost ultimately born by the TSOs (and typically socialized through tariffs). In a paid-as-cleared settlement scheme, one may therefore argue that the optimization objective is the "**payment cost minimization**".
In short, the debate is whether co-optimization should seek to maximize the overall social welfare of both BC and energy bids, or whether it should combine the maximization of social welfare for the energy products with the minimization of BC procurement costs.

There are multiple elements to be considered in such a complex discussion.

Firstly, Art 58 §3 of EBGL [1] states "two or more TSOs exchanging balancing capacity shall develop algorithms to be operated by the capacity procurement optimization functions for the procurement of balancing capacity bids. Those algorithms shall minimize the overall procurement costs of all jointly procured balancing capacity". A straightforward interpretation of this Article is that the algorithmic objective should be "payment cost minimization". Though, it is not completely unambiguous and the alternative "bid cost minimization" is not explicitly ruled out.
On the other hand, Art. 40 of the same guidelines – which specifically targets co-optimization – imposes to compare the "actual market value of cross-zonal capacity" for BC and energy to split the CZC among the different products. CACM guidelines [24], i.e. the key piece of legislation applicable to energy day-ahead markets, unambiguously defines the pricing of day-ahead cross-zonal capacity (Art. 42 of CACM) as "the difference between the corresponding day-ahead clearing prices of the relevant bidding zones", these clearing prices being set while "aiming at maximizing economic surplus" (Art. 38 of CACM).
It therefore yet remains open whether, because BC and energy are governed by different legislative documents, a different methodology should be applied to determine the CZC market value for BC and energy.

Secondly, welfare maximization has been extensively studied by academia and industry, and provides results with desirable economic properties. In particular, economic theory shows that the optimal bidding strategy of a fringe asset under welfare maximization is to bid its true marginal costs, and that the consequent results provide maximal satisfaction to all participants and the highest overall economic efficiency thanks to optimal short- and long-term incentives. To the best of our knowledge, no industry or academic evidence show that the same properties can be obtained through a different objective function such as a "payment cost minimization". Specifically, while a "payment cost minimization" will by definition lead to lower BC procurement costs given a defined set of bids, it remains to be studied if the approach isn't counter-productive on the longer run because (1) BC bids may be adapted because it no longer is theoretically optimal to bid marginal costs and/or (2) the market value being decreased and shared differently under such an alternative objective function, the optimal long-term incentives are no longer effective, which in turn may lead to inefficiencies and potentially higher BC procurement costs.

Thirdly, while welfare maximization naturally leads to desirable price properties (at least in absence of indivisible bids), all (or at least some of) these pricing properties need to be explicitly imposed under "payment cost minimization" (for example, the property that clearing prices equalize in uncongested areas is not naturally satisfied under "payment cost minimization"). This makes the "payment cost minimization" intrinsically more computationally challenging, especially if with a combined objective function setup that also maximizes the energy welfare.

Note that – to the best of our knowledge – all markets that have implemented co-optimization anywhere in the world use a unified objective function based solely on social welfare maximization.

Though, this topic is extremely important given the current TSO practices, the fact that BC procurement costs are socialized, the current legislative stakes and wording, etc. Unfortunately, it has not been fully addressed in the scope of this study given the timing and budget constraints.

This topic is further discussed in Section 5 of this report.

1.6 Network / Congestion rents

It is unquestionable that day-ahead market coupling is an efficient way to allocate CZC to energy markets. For example, under ATC grid model, market coupling ensures that either there is no price differential over an interconnector, or the interconnector is saturated/congested in the direction of the positive price spread. This also results from a non-arbitrage equilibrium: would there be a price difference although no congestion, arbitrageurs would exploit the inefficiency by flowing further in the positive price spread direction, until either the spread vanishes, or until the interconnector is congested.

Full co-optimization provides results satisfying the same non-arbitrage equilibrium principles. Though, the fact that multiple products are auctioned at once – hence that CZC is allocated to multiple products – implies that these equilibrium rules will change.

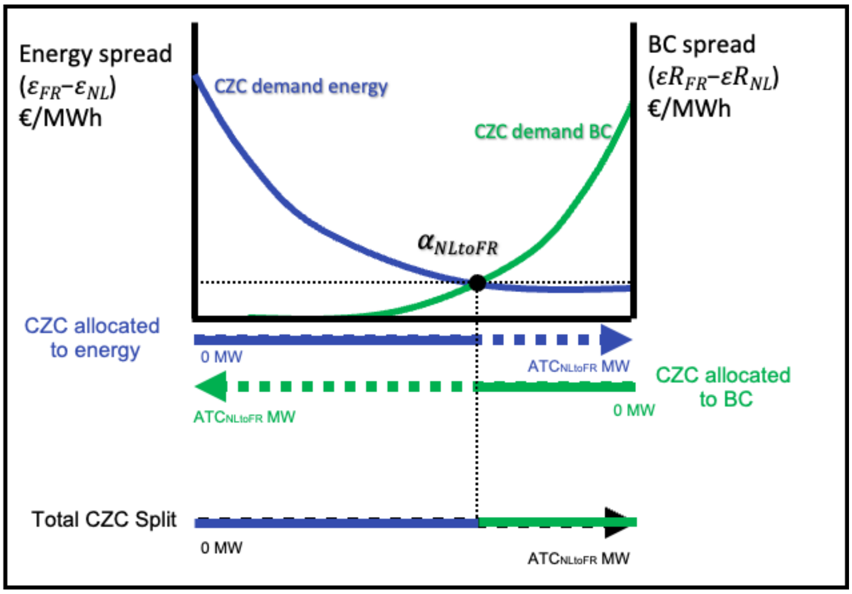

Figure 5: This figure illustrates schematically the equilibrium of a CZC split between BC and energy. The (implicit) demand for CZC stemming from energy – from left to right in blue – is compared with the (implicit) demand for CZC stemming from BC – from right to left in green. The intersection of these curves defines the optimal total CZC split.

Suppose two markets A & B, linked with a single ATC interconnector of 20MW in each direction. Suppose also the following order books:

	Energy market	BC market
Area A	Supply bid 100MW @ 10€/MWh Demand bid 1000MW @ 9€/MWh	Supply bid 1100MW @ 5€/MWh Demand bid 1000MW @ 1000€/MWh
Area B	Supply bid 100MW @ 21€/MWh Demand bid 1000MW @ 20€/MWh	Supply bid 1000MW @ 50€/MWh Demand bid 1000MW @ 1000€/MWh

The co-optimized results for this use-case are the following:

	Energy market	BC market
Area A	Accepted supply: 0MWh Accepted demand: 20MWh Import: 20MWh Clearing price: 9€/MWh	Accepted supply: 1040MWh Accepted demand: 1000MWh Export: 40MWh Clearing price: 5€/MWh
Area B	Accepted supply: 20MWh Accepted demand: 0MWh Export: 20MWh Clearing price: 21€/MWh	Accepted supply: 960MWh Accepted demand: 1000MWh Import: 40MWh Clearing price: 50€/MWh

We note that, on the energy market, Market B clears at a more expensive price (21€/MWh) than market A (9€/MWh). Without co-optimization, the energy would logically flow from market A to market B. However, we observe that the optimal results go in the opposite direction. From an equilibrium point of view, this may be surprising since such an "adverse flow" destructs value (in this case 12€ of value is destructed for each MW flowing from B to A). This is though optimal given flows in a given direction on the energy market free up capacity in the opposite direction due to netting and that, on the BC market, each additional flow from the cheap market A (5€/MWh) to the expensive Market B (50€/MWh) creates more value than what is destroyed by the energy market adverse flows (in this case 45€ of value is created for each MW of BC over the interconnector, which is more than the loss of 12€ induced by adverse energy flows). This example illustrates that, under co-optimization, cross-zonal price differentials in the energy and in the BC markets are fundamentally interlinked with the split of CZC.

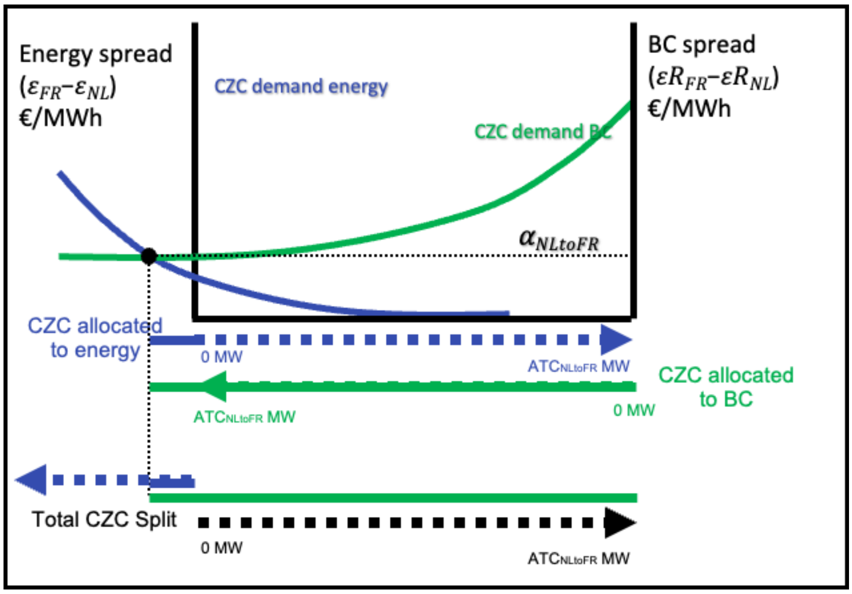

Figure 6: Compared to the previous one, this figure schematically illustrates a case where the intersection between the (implicit) demands for CZC implies "adverse energy flows".

The risk with sequential co-optimization (such as Scenario B2 discussed in Section 4) is that the cross-zonal price properties ensuring coherence of the CZC splitting results no longer apply. This could for example be the case if the final energy prices are recalculated in subsequent steps and differ from the prices calculated during the CZC split (i.e., "volume coupling effects").

Part I: Market Design Analysis

2. Bid linking

As highlighted in the introduction of this document, the linking of bids between the day-ahead and the balancing capacity markets was analyzed as part of the co-optimization of energy and reserve capacity investigation. This subject was first tackled in an Artelys study [2] for which the findings were then detailed in the Implementation Impact Assessment (IIA) [4]. The purpose of bid linking in a co-optimization of energy and reserve capacity context is to allow market participants to better express their technical and economic characteristics while being able to bid in both markets.

The Artelys study [2] and the Implementation Impact Assessment [4] focused on mutually exclusive links which englobe ***multilateral*** and ***unilateral*** links. This type of link allows a production unit to offer a certain power at a certain price for both markets with the restriction that the sum of the accepted power at each market cannot exceed the bided power (mutual exclusive linking in today's SDAC jargon). The difference between unilateral and multilateral only lies in assigning a higher priority to one particular market compared to the other for this bid to be accepted if in the money in the prioritized market. The differences and uses of the two following types of links are detailed and illustrated in the following section of the report. Nonetheless, note that other types of links can be used to encompass the variety of technical and economic requirements of market participants (see Section 8.1). Even though investigating these other types of links goes beyond the scope of the present study, we can cite as an example of these other possibilities parent-child links allowing for example a market participant to only offer downward reserve capacity if being accepted in the day-ahead electricity market.

In the remaining part of this section, as required by the stakeholders, only multilateral and unilateral links will be defined and investigated as the bid linking retained options while considering a unique balancing capacity product (e.g. mFRR up). Some illustrative examples will be used in order to highlight their implications in terms of market design and economic efficiency.

2.1 Multilateral Linking

In the first place, we will focus on the definition and analysis of the mutually exclusive relation called multilateral linking. The explanations below define such a link and illustrate its behavior both in a single zone setup and in a setup with multiple zones coupled through cross-zonal capacities.

2.1.1 Definition and formulation

As mentioned in the introduction of this section, multilateral linking is one form of mutually exclusive linking. A multilateral bid will allow a market participant to bid some quantity in both markets while the sum of the acceptance ratios of his bids in both markets will be at maximum equal to 100%. Using mathematics, this link can be represented by the following constraint: $x + xR \leq 1$ where x and xR represents the acceptance ratio of the bid (going from 0 to 1) respectively for the day-ahead and for the balancing capacity market. The following constraint can be added to the algorithm clearing both markets and operating the CZC split.

In a context where a co-optimization algorithm clears both the balancing capacity and day-ahead market based on welfare maximization, the following properties can be observed with multilateral linking regarding the cross-product linked bid:

- The bid linking constraint is only impactful in case the two linked bids are in (or at) the money. Indeed, if at least one of the two linked bids is not in the money, then the following typical market clearing rules apply:
    - Strictly out of the money orders must be fully rejected;
    - Strictly in the money orders must be fully accepted;
    - Partially accepted orders are at the money.
- If both linked bids are in (or at) the money, then any welfare optimal solution will always select among linked bids the one that is the most profitable. Indeed, if a cross-product linked bid is in the money for several markets it will be awarded only to the market where it generates the highest total surplus. In case of tie, the bid linking constraint can be apportioned between the linked bids.
- The profit of the accepted linked bid always at least compensates for the opportunity loss of the rejected linked bid. Consequently, linked bid selection rule de facto embeds the "opportunity costs" of not being selected in the other market. For example, an asset that has solely a marginal production cost (and no other reservation cost) can therefore bid its balancing capacity at price 0€ with multilateral linking, as long as its energy bid is priced at the marginal production cost. As highlighted in the introduction of this document, this is a useful property of multilateral linking for market participants since the burden of forecasting the day-ahead electricity price in advance to compute their potential opportunity cost is not anymore necessary.

Appendix C illustrates how such properties can be formally proven.

2.1.2 Illustration

In this section, we will now illustrate the different properties of multilateral linking presented hereabove. In that aim, we will first work on an example only considering a single zone. Then, a setup with multiple zones coupled together will be studied.

Single zone setup discussions

To analyze the behavior of multilateral linking in a single zone setup, we will revisit Example 1.1 presented in Section 1.1.

Example 2.1: Illustration of Multilateral Linking for a Single Zone setup

Let us consider a market with the products indicated in Table 5. Note that both assets can only produce 5 MW in total. In other words, the sum of the capacity provided to both markets by each asset cannot exceed 5 MW. The merit orders of the energy and balancing capacity markets are indicated in Figure 7.

Asset A (5 MW)	20 €/MW for BC	25 €/MWh for energy
Asset B (5 MW)	22 €/MW for BC	40 €/MWh for energy
Demand	5 MW BC at any price	5 MW DAE at any price

Table 5: Illustration of Multilateral Linking

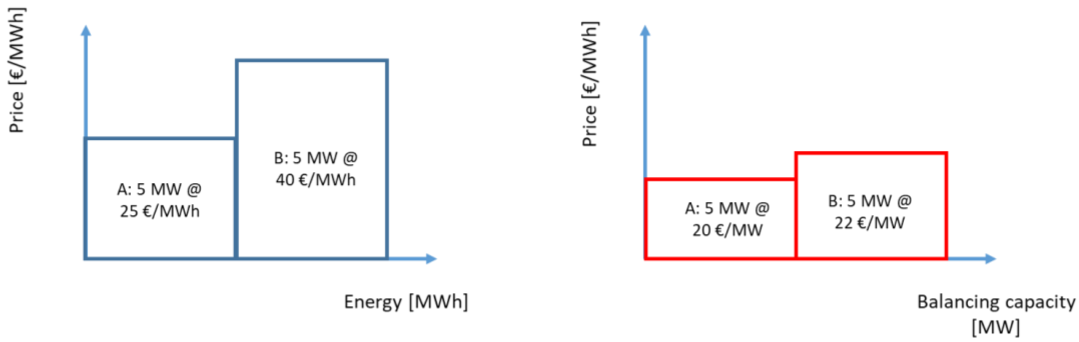

Figure 7: Linked merit orders of the energy (left) and balancing capacity (right) market for Example 2.1.

Suppose that no priority is given to each one of the two markets, i.e. the mutually exclusive link presented above is seen as “multilateral bid linking”. In that case, one optimizes the total welfare generated by the clearing of both markets at the same time. In such a situation, Asset A is selected for the day-ahead market while Asset B is chosen for the balancing capacity market. Indeed, even though Asset A is also the cheapest bid in the balancing capacity market, the gain of assigning Asset B to the balancing capacity are higher regarding both market as a whole since the price of Asset B for energy is high. Therefore, the total sourcing cost in this case is equal to 235 € (22x5=110 € for BC and 25x5=125 € for energy).

Multilateral linking and CZC split

We can now proceed by observing the behavior of multilateral bid linking in a multiple zone setup. We consider here a simple case with 2 zones. Two situations can arise in that setup: the line linking two different zones can either be congested or not. In both cases, the properties mentioned above regarding bid acceptances remain valid. On top of them, the coupling effect explained in Section 1.6 due to co-optimization also applies. Indeed, if the ATC line between two zones is not congested then the day-ahead electricity prices are equal for the two zones. This is also true for balancing capacity prices. However, in the case of a congested ATC line between zones in a given direction, the price differences are such that the cross-zonal capacity is allocated optimally. For example, if the CZC is used for both markets (energy and balancing capacity) in a given direction between two zones then the difference between the day-ahead prices in each zone is equal to the difference between balancing capacity prices in these two zones. However, in the situation where the CZC is only used for the exchange of energy, the CZC value for the exchange of energy is an upper bound on the CZC value for the exchange of balancing capacity in that direction. Note that there are other possible cases, remaining coherent, of allocations of CZC and CZC values for the exchange of energy and balancing capacity. For more details on the subject, an exhaustive list of these configurations along with their implications in terms of prices and CZC values are given in Appendix C of the present report. The conjunction of multilateral bid linking and co-optimization properties are illustrated in the following example.

Example 2.2: Illustration of Multilateral Linking for a Multiple Zones setup

In this example, let us consider the market situation presented in Figure 8. This situation only accounts for a single linked bid in the NL zone.

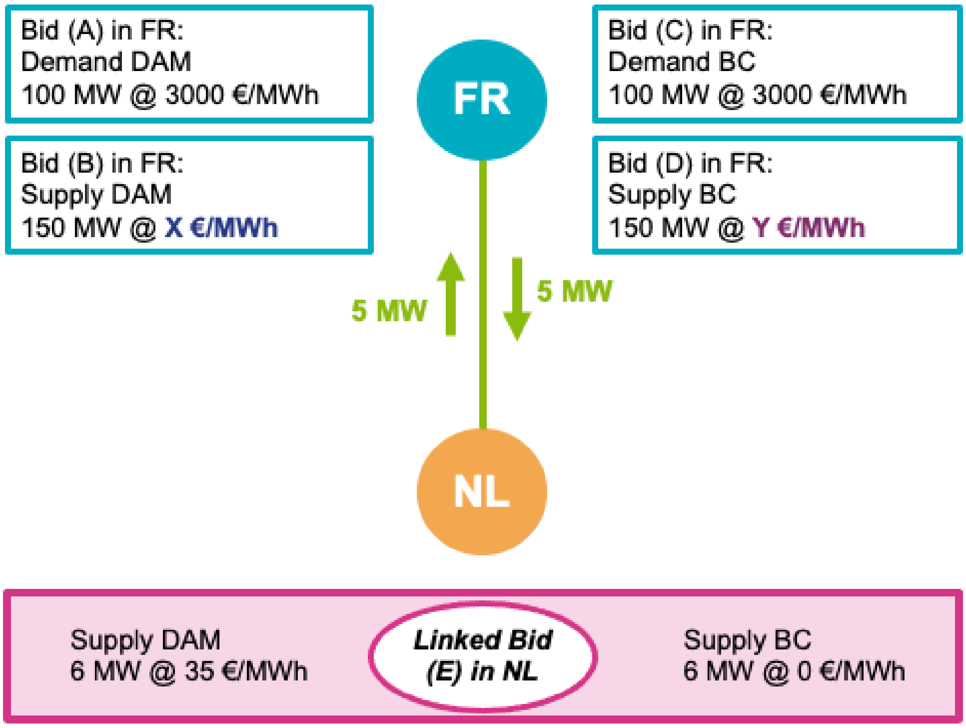

Figure 8: Illustration of multilateral linking in a multiple zones setup (Example 2.2)

The obtained market clearing results for different values of **X** and **Y** are presented in Table 6. From these results, we can observe that even though the balancing capacity portion of the linked bid is proposed at a zero price, it is allocated to the most profitable market. Indeed, by maximizing welfare and determining a market equilibrium, prices will be such that the accepted quantities will be optimal for the selfish maximization problem of the market participant submitting the linked bid. Therefore, if the linked bid is matched for balancing capacity, it means that the balancing capacity price in NL is superior (or at least equal) to the day-ahead market price in NL minus 35. However, if the bid is matched in the day-ahead market then it implies that the day-ahead price in NL minus 35 is greater than (or at least equal to) the balancing capacity price in NL (this is why the opportunity cost is embedded in the link under multilateral linking of bids). Note that if the bid is partially accepted in both markets, then these two quantities are equal.

BC Price FR (Y)	20 €/MWh	5 €/MWh	10€/MWh	20€/MWh
DAM Price FR (X)	50 €/MWh	50 €/MWh	30 €/MWh	500 €/MWh
CZC Split (NL → FR)	BC: 5 MW DAM: 0 MW	BC: 0 MW DAM: 5 MW	BC: 5 MW DAM: 0 MW	BC: 0 MW DAM: 5 MW
Demand DAM in FR (A)	100 MW	100 MW	100 MW	100 MW
Supply DAM in FR (B)	100 MW	95 MW	100 MW	95 MW
Demand BC in FR (C)	100 MW	100 MW	100 MW	100 MW
Supply BC in FR (D)	95 MW	100 MW	95 MW	100 MW
Supply Linked DAM in NL (E)	0 MW	5 MW	0 MW	5 MW
Supply Linked BC in NL (E)	5 MW	0 MW	5 MW	0 MW
BC Price NL	0 €/MWh	-10 €/MWh	0 €/MWh	-445€/MWh
DAM Price NL	30 €/MWh	35 €/MWh	20 €/MWh	35 €/MWh
CZC Value	20 €/MWh	15 €/MWh	10 €/MWh	465 €/MWh

Table 6: Multilateral bid linking in a 2 zones setup: market clearing results for different values of X and Y

2.2 Unilateral Linking

Since the properties and definition of multilateral bid linking has been depicted previously, the other type of mutually exclusive link called "unilateral bid linking" can now be covered. The following section will define the main characteristics of such a link and illustrates its behavior via several examples. We limit our discussions to a setup with solely two markets (energy plus one single upward balancing capacity product).

2.2.1 Definition and formulation

As "multilateral bid linking", unilateral bid linking is a form of mutually exclusive link: a market participant can bid some quantities in both markets meanwhile only being able to provide once this amount of power in total[4]. Therefore, it implies that the sum of the acceptance ratios in each market can at most equal 100%.

The difference between multilateral and unilateral linking lies in the fact that a pre-defined market clearing prioritization is applied under unilateral linking. The intention is that – if a linked bid is proposed to both markets – it should be accepted first in the balancing capacity market. The ideal setup would be that if a linked bid is in the money in a market with high priority, it should be accepted in this market regardless of the potential profits it can make in markets with lower priority. This means that under unilateral bid linking, even if it is more profitable (from a total welfare and/or from an individual surplus perspective) to accept a bid in the energy market, it should in principle be accepted in the balancing market as long as its price on that market is compatible with the clearing price.

In practice, this prioritization is realized by using a sequence of optimization programs as depicted in Figure 9. First, the balancing capacity bids are passed to a box called "BC Attempt Dispatch". In this box, an optimization program is run allowing to clear the balancing capacity markets by accounting for the coupling of zones without considering day-ahead bids. The results of this optimization program provide the acceptance ratio of each balancing capacity bid. Then, these acceptance ratios are passed on to the blue box called "Remove links" along with original balancing capacity and day-ahead bids. During this phase, the linked bids are transformed into two distinct bids. The new balancing capacity bid has the same price but its quantity is equal to its acceptance ratio in the previous step times the original bided quantity. Regarding day-ahead, the linked day-ahead bid is replaced by a non-linked day-ahead bid with the same price but its quantity is equal to its original bided quantity times the available acceptance ratio left of the linked bid after creation of the non-linked balancing capacity bid based on the results obtained at the previous step. For example, imagine that a unilateral linked bid has a quantity of 5 MW and a price of 35€/MWh for day-ahead and 15 €/MWh for balancing capacity. In the scenario of 3 MW of accepted balancing capacity in the first pink box, then the two new bids that will be provided to the co-optimization program will be 2 MW of day-ahead energy at 35 €/MWh and 3 MW of balancing capacity at 15 €/MWh. Finally, as mentioned previously, the modified balancing capacity and day-ahead bids will be submitted to the co-optimization program which will clear both markets based on this new information. Note that in that final step, there is no link anymore between bids of different markets. As it will be illustrated below via examples, while this practical implementation of the unilateral bid linking indeed gives higher priority to some markets over others by design, the properties of the results can hardly be analytically described. In particular, the principal requirement stating that a unilaterally linked bid with higher priority should be accepted if it is in the money – irrespective of its possible value in markets of lower priority – cannot be guaranteed.

[4] We remind that we only discuss in this study bid linking of energy and of a single balancing capacity product. In other words, unilateral bid linking of multiple balancing capacity products has not been assessed in the present study.

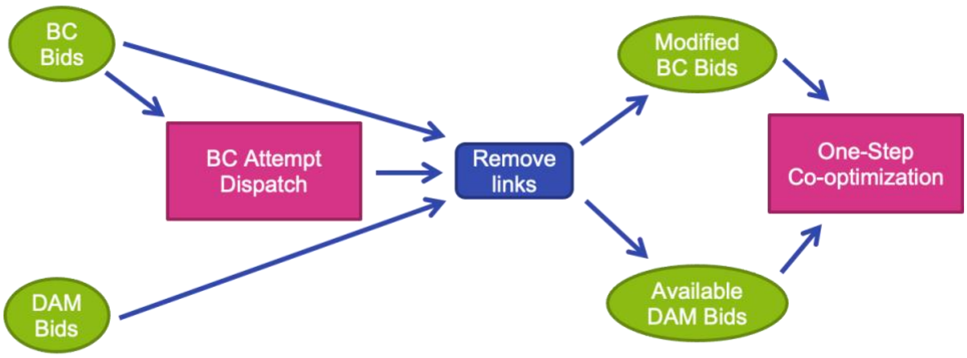

Figure 9: Sequence of operations for unilateral bid linking

In a welfare maximization context for both markets, the properties valid under multilateral linking are not necessarily valid anymore for unilateral bid linking due to the prioritization step. First of all, as highlighted in Section 1.1 and again in Example 2.3 below, unilateral bid linking can lead to loss of efficiency in terms of total welfare combining both markets. Indeed, a linked bid may be accepted in the balancing capacity market even though it would have been more profitable in the day-ahead market which causes the efficiency losses. Furthermore, Section 1.2 shows that the use of unilateral linking can lead to incoherent prices which give wrong incentives to market participants. Finally, as emphasized in Section 1.3, the simplification of bidding behavior for market participants is not possible with unilateral linking. Indeed, the co-optimization program in that case does not have the possibility to embed opportunity cost for market participants. They will therefore have to forecast the day-ahead price. This forecast will generate inefficiencies since it is usually not exact and a small deviation from the true day-ahead price can influence the outcome of the co-optimization algorithm under unilateral linking. Moreover, while the intention of unilateral bid linking is that in-the-money BC bids must always be accepted, this is not always the case in the proposed implementation. Therefore, even if the energy price forecast (and its related opportunity cost) is perfectly forecasted, unilateral bid linking may nonetheless inefficiently allocate a linked bid. This phenomenon will be highlighted in Example 2.4.

2.2.2 Illustration

In this section, the behavior of unilateral bid linking will be illustrated in practice. As realized for multilateral linking, two cases will be used: one with a single zone setup, the other considering multiple coupled bidding zones with co-optimization of energy and reserve.

Single zone setup discussions

Example 2.3: Illustration of Unilateral Linking for a Single Zone setup

Let us extend the considerations made in Example 2.1 by considering now unilateral bid linking for assets A and B (as considered in Example 1.1 in Section 1.1). The considered setup along with the merit orders of the energy and balancing capacity markets are reminded in Table 7 and Figure 10.

Asset A (5 MW)	20 €/MW for BC	25 €/MWh for energy
Asset B (5 MW)	22 €/MW for BC	40 €/MWh for energy
Demand	5 MW BC at any price	5 MW DAE at any price

Table 7: Illustration of Unilateral Linking.

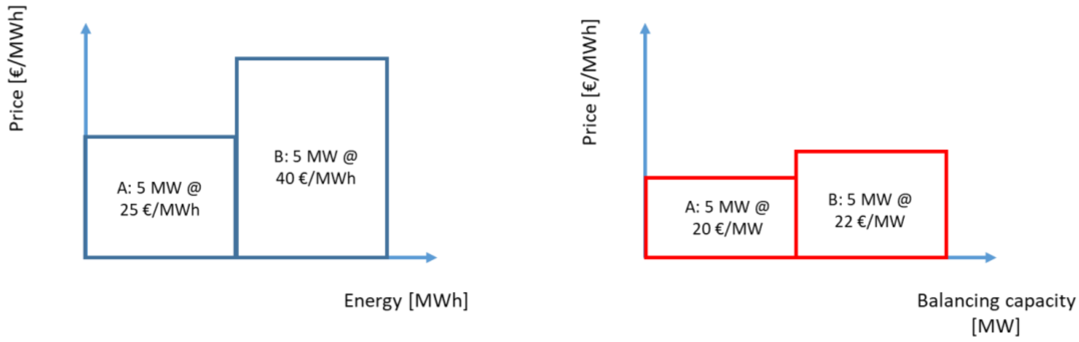

Figure 10: Linked merit orders of the energy (left) and balancing capacity (right) market for Example 2.3.

As explained in the previous section, to apply unilateral bid linking, first the balancing capacity market has to be optimized first, without regards to the energy bids. In such a case, Asset A is selected, because it stands before Asset B in the merit order insofar as balancing capacity only is concerned, as indicated in the left panel of Figure 10. With such a sequential optimization, Asset A is no longer available for day-ahead. Therefore, Asset B is selected for day-ahead at a price of 40 €/MWh. The total sourcing cost in this case is equal to 300 € (20x5=100 € for BC and 40x5=200 € for energy). This example highlights the deterioration of efficiency realized by unilateral linking since in Example 2.1 with multilateral linking, the total sourcing cost amounted only to 235 € (22x5=110 € for BC and 25x5=125 € for energy).

Unilateral linking and CZC split

In this part, we detail Example 2.4 as the unilateral version of Example 2.2. It depicts the behavior of unilateral bid linking with respect to a multiple zone setup.

Example 2.4: Illustration of Unilateral Linking for a Multiple Zones setup

The setup proposed for this example is represented in Figure 11. Note that as explained previously, the balancing capacity linked bid is not proposed at zero price but at a price equal to the opportunity cost of the linked bid. In this case, the forecasted price is 50 €/MWh therefore, the opportunity cost is computed as 50 €/MWh minus 35 €/MWh which is equal to 15 €/MWh.

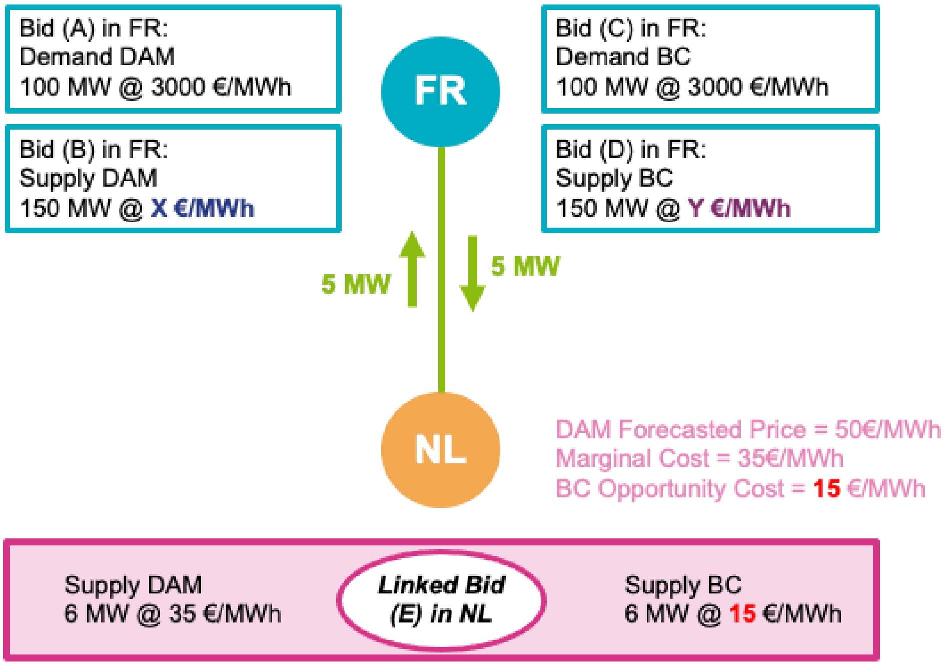

Figure 11: Illustration of Unilateral linking for a multiple zone setup

The results for different values of prices X and Y are given in Table 8. In the first column of this table, the DAM price has been underestimated (50€/MWh estimated vs. 45€/MWh realized). With such prices, the linked bid can make 45-35=10€/MWh of profit per MWh of sold energy. On the BC market, the surplus of the bid is 20-15=5€/MWh, but the real profit is 20€/MWh (since the BC bid price embeds 15€ of opportunity cost and zero actual cost). The bidder is therefore not fully satisfied: given the clearing prices, he would have preferred to sell 1MW more on the BC market and 1MW less on the energy market, as this generates higher profit. In other words, the "BC prioritization" stemming from the unilateral bid linking intention is not fully met.

The reason of this inefficiency is actually not related to the energy price forecast error, but to the actual implementation of unilateral bid linking. Indeed, during the "BC attempt Dispatch" calculation, it is defined that at most 5MW of BC can be sold because of the CZC constraint. As the linked bids are of 6MW, this leaves 1MW of capacity on the energy side. In the next "co-optimization" calculation step", it is thus possible to match at most 5MW of BC and at most 1MWh of energy for these (previously linked) bids. Since (1) this calculation step optimizes the sum of all surpluses, (2) the energy bid creates more welfare/surplus/value than the BC bid and (3) the zone is able to export at most 5MW, the 1MW of energy is matched despite the initial intention to prioritize BC over energy. This inefficiency thus relates to the actual implementation technique of unilateral bid linking, where CZC constraints are fully replicated in the BC "Attempt Dispatch" calculation. However, similar examples can be presented for other implementation variants of unilateral bid linking, for which no perfect and scalable implementation is identified.

BC Price FR (Y)	20 €/MWh	5 €/MWh	10€/MWh	20 €/MWh
DAM Price FR (X)	50 €/MWh	50 €/MWh	30 €/MWh	500 €/MWh
CZC Split (NL → FR)	BC: 4 MW DAM: 1 MW	BC: 0 MW DAM: 5 MW	BC: 0 MW DAM: 0 MW	BC: 4 MW DAM: 1 MW
Demand DAM in FR (A)	100 MW	100 MW	100 MW	100 MW
Supply DAM in FR (B)	99 MW	95 MW	100 MW	99 MW
Demand BC in FR (C)	100 MW	100 MW	100 MW	100 MW
Supply BC in FR (D)	96 MW	100 MW	100 MW	96 MW
Supply Linked DAM in NL (E)	1 MW	5 MW	0 MW	1 MW
Supply Linked BC in NL (E)	4 MW	0 MW	0 MW	4 MW
BC Price NL	15 €/MWh	-10 €/MWh	10 €/MWh	15 €/MWh
DAM Price NL	45 €/MWh	35 €/MWh	30 €/MWh	495 €/MWh

Table 8: Results for the example of Unilateral linking for a multiple zone setup

3. Flow-Based Compatibility and the Deterministic Requirement

In this section, we discuss options that are considered for enforcing the flow-based compatibility requirements. This is also referred to above, and in the N-SIDE/Afry study [3], as the deterministic requirement. Alternative probabilistic flow-based compatibility requirements were also considered in [3], however they are not treated further in the present report. The deterministic requirement is discussed in the present section as a separate topic, since the ways to enforce it apply to *both* the co-optimization option (Scenario A1) as well as the sequential optimization options (Scenario A2 and Scenario B2)[5].

3.1 Definition of the Deterministic Requirement

As the level of activation of the TSO demand in real time is not known in advance, a key question is the following. Is it possible to ensure that, for any activations of TSO demands lower than the TSO demands that are matched in balancing capacity market clearing, the network can support the resulting flows that are required for balancing this configuration of TSO demands?

[5] Scenarios are described and discussed in Section 4.

The deterministic requirement can be stated in a semi-formal manner as follows: for any solution (x^*, xR^*, n^*, nR^*)[6] satisfying all market constraints, including network constraints, and for any $xR_i^\# \leq xR_i^*, i \in IR: QR_i > 0$ (i.e., any lower levels of TSO demand activation in real-time), one can find $xR_i^\# \leq xR_i^*, \in IR: QR_i < 0$ (i.e., lower levels of supply BC bid activations in real-time), and $nR^\#$ (i.e., resulting BC net positions) such that $(x^*, xR^\#, n^*, nR^\#)$ remains feasible for the network constraints.

3.2 Enforcing the Deterministic Requirement: Challenges and Ways Forward

The deterministic requirement is approximated in [3] as a stochastic program, where we consider the corner cases of TSO demand activation (zero and full activation). It is also expressed in [3] as a robust optimization problem. Both the robust optimization and the stochastic programming formulation are computationally intractable. Moreover, it is important to emphasize that no such requirement is represented in market clearing models in other worldwide markets[7]. The deterministic requirement is seen to be particularly complex as it amounts to a robust optimization problem with decision-dependent uncertainty, which escalates the complexity of the already extremely challenging class of robust optimization problems with decision-independent uncertainty [12].

In order to gain intuition about why the problem is hard, note that in order to satisfy the deterministic requirement we are in principle required to consider 2^N combinations of TSO demand realizations for N TSO demands that are matched in the market. And this is simply considering a single BC product and a single period. If we consider three BC response speeds (aFRR, mFRR, RR), each with an upward and a downward direction, this amounts to 6 BC products per time period. With 96 time periods in future market-clearing models, this implies $2^{96 \cdot 6 \cdot N}$. Even if we had a handful of TSO demands (we obviously have much more in a realistic instance of EUPHEMIA), the number of combinations to consider is clearly prohibitive. Unlike for binary constraints, where extensive scientific/mathematic research has produced very efficient algorithm to resolve such exponentially difficult problems (e.g., branch&bound/cut), no similarly efficient approach exist to resolve stochastic or robust optimization programs.

It is important to bear in mind that this requirement needs to be added on top of an already challenging problem, i.e., a large-scale mixed integer quadratic program subject to complementarity constraints, which needs to be solved within a few minutes. However, as indicated in the previous paragraph, the problem here is not so much the existing mixed integer quadratic program subject to complementarity constraints, which is the current formulation of EUPHEMIA. Instead, the deterministic requirement appears to be by nature intractable (and

[6] Here, x^* denotes the energy bid acceptances, xR^* the balancing capacity bid acceptances, n^*, nR^* are respectively the resulting energy and balancing capacity net positions, IR is the set of BC bids, and QR_i the bid quantities (positive for demands, negative for supplies).

[7] Whereas US market clearing models include security constraints, they do not include a deterministic requirement.

this is corroborated by the fact that no such requirement exists in other market clearing models worldwide).

Over the past months, and since the original study on co-optimization [3], our team has investigated a range of alternatives for tackling this extremely challenging requirement. In our original work [3], we formalized the problem as a robust optimization problem, and subsequently proposed a stochastic programming approximation. A decomposition algorithm based on the academic literature [13] was considered, but was not found to be effective for the problem. A fallback conservative approximation was proposed for furnishing a solution to the problem within an acceptable run time. Through tests on illustrative case studies, these alternatives were found to be either non-scalable or too conservative for the needs of a realistic instance.

More promising innovations were proposed by the NSIDE team after the conclusion of the original study [3] and following internal R&D, some of which is summarized in [15] and continued in the present report. These innovations are inspired by existing practices that are applied for the computation of ATCs for intraday markets, as well as know-how related to the formation of intuitive cuts in EUPHEMIA [14]. The general geometric intuition of these approaches is to approximate various key geometric objects of the problem (e.g., the set of feasible net injections of the model) by boxes. Boxes are easier to handle computationally, in particular it is easier to inscribe boxes within more computational geometric objects by simple arithmetic operations [16] (e.g., by replacing the coefficients of certain variables by their non-negative part). The intuition is that when we are trying to inscribe a box in a more complicated shape, we do not need to consider the combination of corners of the box, but only the "hardest" corners, which are obtained by replacing the coefficients of certain variables by their non-negative part.

We summarize the outcome of our investigation in Table 9. We first summarize the options that we have considered, and we then comment on the evaluation criteria that we have used in order to rank these options:

- **SP**: the stochastic programming approximation of the deterministic requirement. This approximation is possibly exact for problems with well-behaved market products (e.g. curves), but can be shown to be inexact for more complex market products (e.g. block orders).
- **Decomposition method of [13]:** This algorithm has been considered for application in US day-ahead market clearing models, at the stage of residual unit commitment. It turns out that these formulations are simpler than our requirement, because they do not involve decision-dependent uncertainty [12]. This approach was not considered in further detail, following disappointing initial results in simple instances [3] of our problem.
- **Conservative approach of [3]:** This was proposed as a fallback to [13] in [3]. It is, however, easy to show that this approach can be expected to be overly conservative and

limit excessively efficiency of trade in the BC market. This option is therefore not considered further in the present report.

- **Inscribed boxes of net BC supplies / demands (O1)**: This approach was proposed in [15]. The idea is to inscribe boxes to the polytopes of feasible net supply positions and net demand positions. The approach is discussed further below.
- **Inscribed boxes of net BC injections (O2)**: This approach is proposed in the present report as a way of overcoming a number of weaknesses that were uncovered with respect to (O1). The idea is to inscribe boxes to the polytopes of feasible net injections. The approach is discussed further below.
- **Intuitive cuts (O3)**: This approach is proposed in the present report as a way of overcoming a number of weaknesses that were uncovered with respect to (O1) and (O2). The idea is to characterize the union of ATC polytopes that can be contained in the flow-based polytope of the problem. The approach is discussed further below. For reasons that are discussed in detail in the sequel (but already indicted in color coding in Table 9), **<u>we propose this as the target model for approximating the deterministic requirement</u>**. The approach is discussed further below.

	Tractability	Hub dependence	Conservativeness	BC netting (uniqueness of BC prices)
SP	--	++	++	??
Decomposition method of [13]	-	++	??	??
Conservative approach of [3]	++	++	--	++
(O1) [15]	++	--	0	--
(O2)	++	-	0	++
(O3)	++	++	+	++

Table 9: Alternatives considered for tackling the deterministic requirement, and their ranking according to a number of criteria.

We consider the following criteria when deciding on which approach to target for implementation.

- **Tractability**: The tractability criterion aims at determining to what extent the proposed approach can scale to the realistic scale of the EUPHEMIA instances, bearing also in mind the available time that is afforded to the algorithm and the already existing challenging requirements.
- **Hub dependence:** The hub-dependence criterion aims at determining to what extent the proposed model is invariant to the choice of hub zone (also called slack node) in the physical model. Hub independence is a good thing, and hub dependence is a bad thing. If our market clearing solution depends on which zone we select as the hub for our power transfer distribution factors, we arrive to a paradox where a parameter setting which produces physically equivalent models actually generates market clearing

solutions with different levels of welfare / order matchings / market clearing prices. We thus wish to have a model which is hub-independent.

- **Conservativeness:** It is possible to establish certain relations between the feasible regions of the models, and thus the extent to which they are conservative in approximating the true deterministic requirement (or its stochastic programming approximation). We generally seek less conservative models, so as to enhance trading market welfare and allow more trades without compromising network security.

- **Uniqueness of BC prices:** Netting of supply and demand is a property of economic models which allows us to derive unique equilibrium prices for the same product in the same location. Netting of supply and demand means that one unit of supply of a product and one unit of demand of the same product is equivalent to zero supply and demand of the product, i.e. that the supply and demand cancel each other out. If netting does not apply, it may turn out that the price for supply of one unit of a product may not be equal to the price of demand of one unit of a product in the same location, which is counterintuitive. We therefore favor netting and uniqueness of prices in the same location.

Given the aggressive implementation timeline, we have made choices in terms of which solutions to prototype and consider further. As mentioned already in the introduction of this section, our recommendation below is equally applicable to co-optimization (option 1) as well as sequential optimization (option 2). In summary, **we favor option (O3) due to its universally superior performance in terms of all the aforementioned criteria**. In arriving to this conclusion, we have had to better understand and closely analyze also options (O1) and (O2). Each of these approaches has certain key weaknesses[8], and (O3) appears to overcome them.

We develop the rationale of our recommendation in detail below. We conclude this section by attempting to illustrate the geometric intuition of the different approaches through an illustrative example.

3.2.1 Option (O1)

Option (O1) fits a box in the polyhedron of BSP supplies that the network can support, and another box in the polyhedron of BSP demands that the network can support. It turns out that this model has a less restrictive version, (O2), which can be shown to be sufficiently conservative so as to respect the deterministic requirement. Thus, the orders that are matched from this model can be shown to respect the deterministic requirement as well. This is already a very promising starting point relative to the alternatives that were considered in [3].

Due to its conservative nature, when considering a trade of BC supply and demand at a given location, the model attempts to figure out the most stressing way to ship that BC supply from the location in question to the hub of the system, and from there back to the location where this

[8] Namely hub dependency, conservativeness, and for O1, non-uniqueness of BC prices in a same location.

BC is to be consumed. This means that BC supply and demand in the same location do not cancel each other out, because in concluding a trade these offers use up network capacity, even if they are located in the same zone! The consequence is different prices for BC supply and BC demand in the same location (with BSPs being paid less than what TSOs pay for BC at the same location). Model (O1) thus poses certain institutional challenges, and counter-intuitive and somewhat suboptimal economic properties.

The model also fails to respect hub independence. We do not expand on this discussion for the time being, but rather reserve it for the next section, where we summarize option (O2).

3.2.2 Option (O2)

Option (O2) is inspired by the non-uniqueness of BC prices that is pointed out in relation to (O1). The idea is to inscribe a box to the space of admissible BC net injections, as opposed to treating BC supply and BC demand separately. It turns out that this model satisfies BC netting of demand and supply, leading to unique BC prices per location. It is also less conservative than option (O1): one can establish a procedure for constructing a feasible solution for (O2) from any given feasible solution of (O1). What is most promising is that this option can be shown to satisfy the deterministic requirement in the sense expressed by the stochastic programming formulation of the model (see appendix D of [3] for this stochastic programming approximation). In particular, we established a procedure for constructing a feasible solution to the stochastic programming formulation of the deterministic requirement from any feasible solution of (O2). We thus arrive to a model that can meet the deterministic requirement in a (typically) less conservative way than the conservative approach of [3], but also in a computationally tractable way. This approach also produces a unique BC price per zone, which is economically intuitive and institutionally manageable.

Unfortunately, it turns out from experimental tests that option (O2) is sensitive to which zone we assume to be the hub when constructing the flow-based constraints of the model. This property, which we refer to as **hub dependence**, means that for different equivalent descriptions of the underlying physical model of network constraints we may get different market clearing solutions based on the choice of hub. This property contradicts the fundamental fact that a physical network model is invariant to the choice of hub, and is thus a source of concern. This behavior is also true for (O1). Although experimental evidence suggests that (O2) is less sensitive to the choice of hub, it still is sensitive to the choice of hub (see Table 15 in Appendix B, e.g., the results on Example E1 for different choices of hub). This remaining difficulty motivates formulation (O3).

3.2.3 Option (O3)

Option (O3) is inspired by the "intuitive cuts" procedure that is used for obtaining ATC capacities in the intraday market, as well as the "intuitive cuts" that can be generated by

EUPHEMIA in case so-called "non-intuitive flow-based results" are to be prevented[9]. An important subtlety of the formulation is that the space in which the boxes are inscribed is no longer the space of net injections, as in formulations (O1) and (O2), but rather the space of feasible flows. We have established that this model is *hub independent*. Moreover, the model can also be shown to satisfy the deterministic requirement, and also produces unique BC prices per zone. Empirical evidence provided in Table 15 in Appendix B suggests that it may be less conservative than (O1) and (O2), though we have not provided any formal results in this respect. The method thus satisfies all the necessary requirements that are established in Table 15, **and is thus our recommended target model for co-optimization (Scenario A1) and sequential optimization (Scenarios A2 and B2)**.

It is however not excluded at this stage that the option (O3), or approaches building on similar ideas, can be further refined to further decrease conservativeness. For example, it is still an open question whether a tractable dynamic procedure can be designed, that would dynamically add constraints to enforce the deterministic requirement only when it is not satisfied by a candidate solution of the market clearing problem.

3.2.4 Illustration on a simple example

In order to gain some geometric intuition about the formulations, let us consider the feasible set of Example 5.6 of [3]. The example was originally compiled in order to illustrate the possibility that the deterministic requirement can cause congestion in both directions of a link even in the presence of a single (e.g. upward) balancing product, but is also a helpful instance for gaining geometrical intuition since it involves energy only and is low-dimensional, thus the feasible set of the problem can be visualized. The example is recalled in Figure 12.

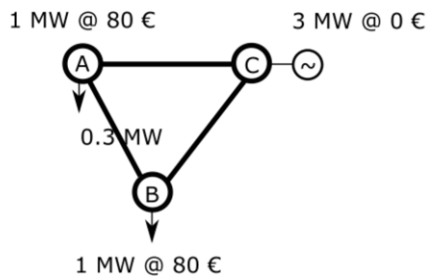

Figure 12: Example 5.6 of [3]. The optimal matching in this example is to allocate 1.8 MW of BC supply to cover 0.9 MW of TSO demand in zone A and 0.9 MW of TSO demand in zone B.

We present the feasible set of the SP formulation in net injection space in Figure 13. To derive the feasible set of net injections, we need to consider the possibly cases of TSO demand activation.

Case 1: both TSO demands are fully activated.

[9] This feature has been used since May 2015 (launch of CWE Flow-based) until October 2020.

$$\frac{1}{3}nR_A - \frac{1}{3}nR_B \leq 0.3$$
$$-\frac{1}{3}nR_A + \frac{1}{3}nR_B \leq 0.3$$

These are the two diagonal purple lines in the Figure 13.

Case 2: TSO demand A is fully activated and TSO demand B is not activated.

$$\frac{1}{3}nR_A \leq 0.3$$
$$-\frac{1}{3}nR_A \leq 0.3$$

These are two horizontal hyperplanes that intersect the vertical axis respectively at -0.9 MW and 0.9 MW (the latter is not presented in the figure, because it is dominated by the requirement that matchings are non-negative).

Case 3: TSO demand A is not activated and TSO demand B is full activated.

$$-\frac{1}{3}nR_B \leq 0.3$$
$$+\frac{1}{3}nR_B \leq 0.3$$

These are two vertical hyperplanes that intersect the horizontal axis respectively at -0.9 MW and 0.9 MW (the latter is not presented in the figure, because it is dominated by the requirement that matchings are non-negative).

Case 4: For the case where none of the TSO demands is activated, the flow-based constraints are trivially satisfied and impose no additional constraints to the problem.

This feasible set is presented in the following figure with shaded purple. The red dot indicates the optimal solution of the problem. Note that the full TSO demands are not matched (at 1 MW): even though matching them would not result in an overload of line AB in the base scenario, it would indeed cause a line overload under cases 2 or 3.

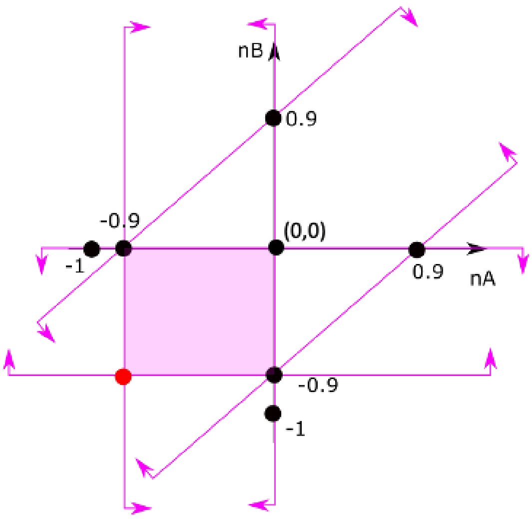

Figure 13: Feasible set of SP for example 5.6 of [3].

The shape of the feasible region already indicates to us that option (O2) will be capable of replicating the feasible solution. The reason is that the purple region is a box in net injection space. And what (O2) attempts to do geometrically is to fit a box that includes the origin of the axes (i.e. the point (0,0)) while trying to cover as much of the lower left space of the figure as possible, since it is in this direction that the objective function of the auction model improves. And indeed the best inscribed box that we can pick is the purple shaded region itself. It does indeed turn out that model (O2) attains the same objective function value as SP for this example, as also reported in Table 15 in Appendix B. This is a fortunate coincidence for this specific example, but is not guaranteed to be the case in general. Model (O2) is generally a conservative approximation of the deterministic requirement. Similar geometric interpretations can be developed for (O1) and (O3), that are also conservative approximations of the deterministic requirements, but at least tractable ones. A geometric intuition of the difference between model (O1) and model (O3) is given on Figure 14. Moreover, (O3) is accompanied by a number of other institutionally desirable properties, as summarized in Table 9, and for this reason it is recommended as the target model for the implementation of the deterministic requirement.

Figure 14: Geometric intuition for models (O1) and (O3): (O1) describes all possible "boxes" in the flow-based domain described with demand and supply net position variables, while (O3) describes all possible "boxes" in the flow-based domain described with cross-border flow variables.

Note that model (O3) is conservation in the sense that it ensures that there also exists an ATC domain within the cross-zonal capacity domain allocated to the balancing capacity markets which satisfies exactly the deterministic requirement. This means that it is always possible to extract from the CZC allocated to balancing capacity an ATC domain for the balancing energy markets such that any combination of activations of the procured balancing capacity is feasible. This has two important consequences:

- the O3 model is guaranteed to be compatible with balancing energy market platforms relying on ATC network models (which is expected to be the case at least during the first years of MARI & PICASSO);
- if the balancing energy market platforms are not based on ATC network models, it is nonetheless guaranteed that the energy balancing prices of these platforms are intuitive (in the sense that the merit order is not "weighted" by the relative impacts of the zones over the grid constraints).

As these two features are likely desirable at least to some extent, and as it can be shown that the O3 model provides the optimal (i.e. non-conservative) CZC allocation if these two features are required, the conservativeness of the O3 model potentially becomes an advantage.

3.2.5 Compatibility of Option O3 with the Extended LTA Inclusion in Euphemia

The Extended LTA Inclusion (ELI) is a mechanism implemented in Euphemia 10.5 by which the initial flow-based domain is enlarged to ensure that the network model includes a so-called LTA domain, to ensure that congestion rents are larger than liabilities towards owners of Financial Transmission Rights. One natural question is to which extent this mechanism is compatible with a co-optimization context where the deterministic requirement is enforced via option (O3). A preliminary analysis, to be further confirmed by a formal proof, suggests that the Extended LTA Inclusion mechanism applicable to energy markets could be maintained in such a way that (a) the congestion rent adequacy is still guaranteed, (b) option (O3) still enforces the deterministic requirement when combined with ELI.

4. Design options envisioned by ENTSO-E

As mentioned in the premise of this document, three designs were requested to be implemented to embody the co-optimization of energy and reserve. In every design option, one can find the three following functions realized either simultaneously in the same optimization program or sequentially:

- **CZC split**: the split and allocation of part of the cross-zonal capacity to either the balancing capacity or the day-ahead energy market.
- **BCM clearing**: clearing of the balancing capacity market
- **DAM clearing**: clearing of the day-ahead electricity market

Each design is given by the combination of two aspects: the linking of bids and the number of steps taken. Regarding the linking of bids, each scenario considers only one type of bid linking namely ***multilateral*** or ***unilateral***. These two types of links were already presented and analyzed in depth in section 2 of this document. In addition to the considered linking, each design contains a procedure with ***one*** or ***two steps***. The difference between one step and two steps lies in the use of Euphemia in a second stage to clear the day-ahead energy market as it is done in Europe currently. These three scenarios consist in:

- **Scenario A1: 1-step multilateral linking approach**
- **Scenario A2: 1-step unilateral linking approach**
- **Scenario B2: 2-step unilateral linking approach**

A detailed explanation of each of these three different scenarios is provided in the following respective sections. The comparison of the different outcomes of each scenario is then provided on illustrative examples at the end of this chapter.

Let us note that the deterministic requirement has been enforced in all of the three scenarios using approach O3 (cf. the overview of approaches to enforce the deterministic requirement in section 3). In relation to recent positions expressed by some market stakeholders, it is important to distinguish co-optimization from the deterministic requirement. These are two different things, and they do not need to co-exist. On the one hand, co-optimization optimizes the allocation of limited generation resources and limited CZC to energy and balancing capacity, and thus de facto improves welfare relative to a suboptimal sequential allocation. On the other hand, the deterministic requirement is a way to cope with the allocation of CZC for balancing capacity, for which actual cross-zonal flows are uncertain at the time of CZC allocation (because – unlike for energy - balancing energy is only firmly activated after CZC allocation, i.e. close to real-time). The deterministic requirement has been imposed on co-optimization, but is not a generic attribute of co-optimization and can equally well be imposed to any cross-zonal procurement mechanism of balancing capacity. Similarly, there exist multiple markets around the world where co-optimization has been implemented under flow-based grid constraints without such a deterministic requirement, for example in the USA.

The type of robustness brought by the deterministic requirement has a cost in terms of computational complexity. From a welfare perspective, the requirement, or how it is implemented, can turn out to be overly conservative, though welfares with and without the deterministic requirement cannot be directly compared, as robustness of the solution against any possible balancing capacity activation is guaranteed in one case and not in the other.

Finally, let us note that welfare maximization or equivalently bid-cost minimization has been considered in these scenarios regarding the procurement of balancing capacity. Section 5 further compares welfare maximization to procurement cost minimization, highlighting the main challenges and open questions related to (marginal price) procurement cost minimization for the procurement of balancing capacity.

4.1 Scenario A1: One-Step Multilateral Linking

First, in this section, we deep-dive into the baseline co-optimization option, scenario A1, which tackles the full set of requirements, including the introduction of reserve products to the energy auction, the deterministic requirement, and the business rules that apply to the pricing of network access and energy/reserve products. The process diagram of this 1-step approach is presented in Figure 15. This process is called one-step multilateral linking as it only accounts for multilateral bid linking as presented in Section 2.1, and since both the day-ahead and the balancing capacity market are cleared at the same time using the same optimization program.

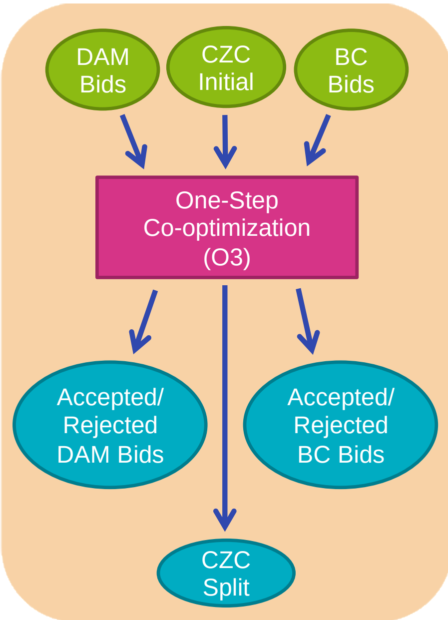

Figure 15 - Process diagram of Scenario A1 (1-step multilateral linking)

The process diagram assumes a tight integration of TSO and power exchange operations. Concretely, energy and balancing capacity products are provided as input to the algorithm, together with "data tags" which link the balancing capacity and energy products in case of bid linking. The initial cross-zonal capacity is also provided to the algorithm. Then, a single multi-production auction model represented by the pink box is solved by a single optimization run and produces coherent prices (for energy, balancing capacity and cross-zonal capacity). Since the model accounts endogenously for the complex interactions between energy, balancing capacity, and cross-zonal capacity, the result which is produced maximizes economic efficiency of trade and produces coherent price signals while accounting for the full set of requirements (incl. flow-based compatibility and integration of bid linking).

Note that this auction produces an outcome that maximizes the economic benefit of all involved market participants. Indeed, as already highlighted in Section 2.1, multilateral bid linking does not alter the property that the auction results will maximize the economic profit of market participants (in the sense that all participants receive maximal profit given their bids and the clearing prices[10]). This property is also not altered by the addition of the deterministic requirement: EUPHEMIA ensures that the bid matchings and prices is of maximum satisfaction to the bidder, and the market, by design. Co-optimization generalizes these market-clearing economic properties in a multi-product setting, i.e. to the selling of not only energy but also balancing capacity. As illustrated below, such a property is not guaranteed by the design of the two other scenarios presented in the next sections.

4.2 Scenario A2: One-Step Unilateral Linking

In this section, the second scenario, namely Scenario A2, is investigated. Figure 16 shows the process diagram of this scenario. It corresponds to a practical one-step implementation of unilateral linking as presented in Section 2.2. This scenario realizes a prioritization of balancing capacity compared to energy in the allocation of linked bids. Note that even though two optimization programs are used in this scenario, it is labelled as a one-step scenario because it will be calculated within a single process (comprising multiple calculation steps). Note also that even though two optimization programs are required to embody unilateral linking, the flow-based compatibility requirement is enforced identically in both mathematical programs.

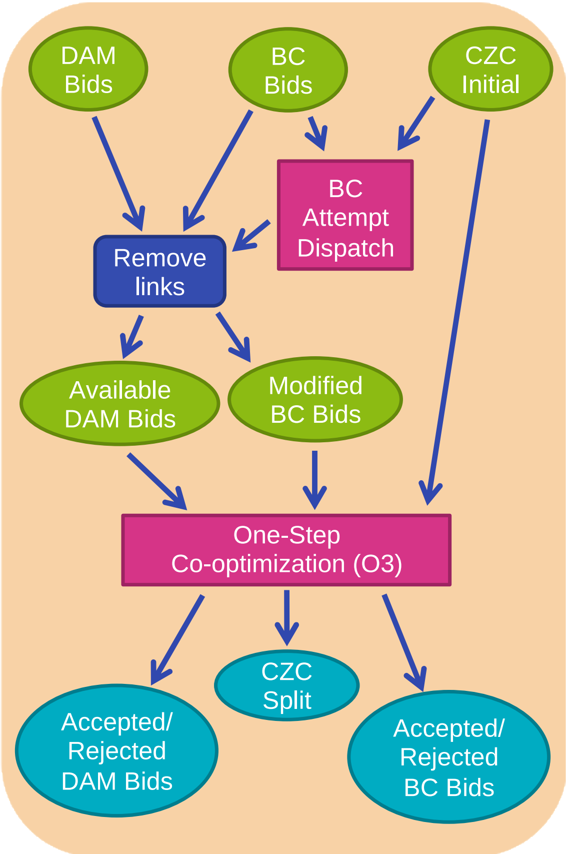

Figure 16: Process diagram of Scenario A2 (1-step unilateral linking approach).

[10] This is excluding the considerations of (non-)uniform pricing and related "paradoxical rejection of blocks" caused by the presence of indivisible bids – which is a distinct topic not addressed in this study.

As presented in the process diagram, a first optimization problem is solved considering only balancing capacity bids without any attention to day-ahead bids. This model is symbolized by the first pink box called “BC Attempt Dispatch” and allows to virtually clear the balancing capacity market alone.

Then, the acceptance ratio of each linked balancing capacity bid resulting from this calculation are passed on to the blue box called “Remove links” along with the original set of day-ahead and balancing capacity bids. The purpose of this step lies into the transformation of linked bids into two separate bids (one for balancing capacity and the other for energy). The transformation is realized by creating a new balancing capacity bid at the same price as the initial linked balancing capacity bid, but possibly a volume constrained by the acceptance ratio obtained at the previous step. Similarly, a new energy bid is created at the same price as the initial linked energy bid, but with a volume possibly constrained by the residual acceptance ratio (i.e. one minus the acceptance ratio obtained for the linked capacity bid at the previous step).
After this computation, this new set of bids and all other existing (i.e. non-linked) energy and balancing capacity bids are provided to the other pink box containing the true co-optimization function allowing to clear both the balancing capacity and the energy market along with allocating the cross-zonal capacity to each market. Note that in that latter step no linking of bids is considered anymore, but the flow-based compatibility/deterministic requirement is enforced.

As already emphasized previously, one-step unilateral linking can lead to several inefficiencies from a market design perspective, that are detailed in Section 2.2. Specifically, and as already explained, one of the major drawbacks of one-step unilateral linking is the fact that - in some particular cases – none of the linked bids are accepted though at least one linked bid is in the money (i.e. is compatible with the clearing prices).

4.3 Scenario B2: Two-Step Unilateral Linking

We now proceed with another version of unilateral linking considering two different steps. The process diagram of Scenario B2 is depicted in Figure 17. Two distinct steps (or processes) are considered in this scenario: the first step highlighted by the orange box can be realized by another IT infrastructure than the second step (this latter step being a run of Euphemia, the current clearing SDAC algorithm). Note that, as for the previous scenarios, the flow-based compatibility requirement is enforced in all relevant optimization programs.

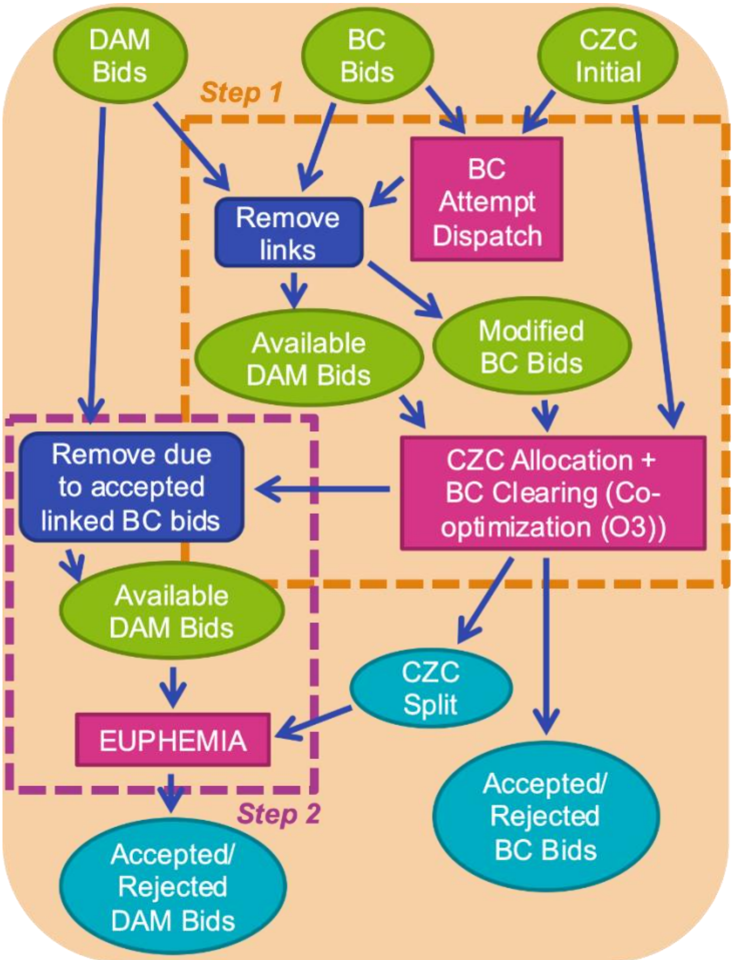

Figure 17: Process diagram of Scenario B2 (2-step unilateral linking approach).

As previously emphasized, this scenario contains a sequential optimization of balancing and cross-zonal capacity (accompanied with a cross-zonal capacity split), followed by an optimization of solely energy. By looking at the diagram of this scenario, we observe that the first step of this computation highlighted by the orange box is the same as Scenario A2 detailed in Section 4.2. The only difference lies in the simplification of several characteristics of day-ahead bids to reduce the complexity of the problem and possibly improve performances. This simplification consists for example in the transformation of indivisible day-ahead bids into divisible ones. The precise simplifications implemented in the prototype of this scenario are given in Section 6.1. Note that only balancing capacity accepted and rejected bids along with the cross-zonal capacity split between markets are fixed at the end of that first step. The results of the middle pink box are then passed on to the blue box called "Remove due to accepted linked BC bids". In this box, the linked balancing capacity quantity that was not accepted during the first step is made available again for energy. For example, consider a unilateral linked bid of 5 MW. After the "BC Attempt Dispatch" calculation, the BC clearing considers 3 MW for balancing capacity (hence 2 MW are in principle left for energy). However, it is possible that in the "CZC allocation and BC clearing (co-)optimization" only 1 MW of this balancing capacity bid is cleared. Therefore, an energy bid of 4 MW is passed on to the last step of the scenario (instead of only considering an energy bid of 2 MW as would be the case in the Scenario A2). Finally, the last step of this scenario is given by the output of a usual run of the EUPHEMIA algorithm considering the new set of day-ahead energy bids (i.e. where linked bids have become simple energy bids after deduction of the cleared balancing capacity part) and the remaining cross-zonal capacity (after deduction of the cross-zonal capacity allocated for balancing capacity). This optimization program thus clears the energy day-ahead market and provides the entire set of results related to energy.

Unlike for scenario A2, this approach prevents linked bids to be rejected if they are in the money in both markets (because there is no reason that this happens in the second step). However, it may lead to infeasibilities as it will be illustrated in the following section. Note also that since balancing capacity and energy are cleared sequentially, the day-ahead and balancing capacity clearing prices have no imposed coherent relations as it was the case for one-step scenarios (i.e., "volume coupling effect"). This latter point is highlighted in Example 4.1 in the following section.

4.4 Designs Comparison on Illustrative Examples

In the interest of analyzing the behavior of each presented scenarios and understanding their differences, we provide several illustrations inspired by discussions that have emerged from exchanges with the stakeholders throughout the project. The following examples illustrate market design challenges related to Scenarios A2 & B2 (i.e. involving unilateral bid linking).

- Example 4.1 shows that unilateral bid linking can lead to **welfare losses, and that a 2-step setup can lead to incoherent prices.**
- Example 4.2 illustrates how unilateral bid linking can lead to a linked bid being fully rejected in all markets although it is in-the-money in the energy market.
- Example 4.3 shows that in the Scenario B2 (2-step scenario with unilateral bid linking), the split of the co-optimization run into two steps can lead to situations where step 1 results make step 2 calculations infeasible.

Example 4.1: Welfare losses for unilateral liking and imperfect price equilibrium in a two steps setup

Consider the following example detailed in Figure 18. In this case, two bidding zones are considered (FR and NL). Each bidding zone contains supply and demand bids for the balancing capacity and the day-ahead markets. An ATC line is connecting both regions with a capacity of 5 MW. A single linked bid is proposed in Netherlands representing an entity with a marginal cost of 35 €/MWh and with a maximum production of 6 MW.

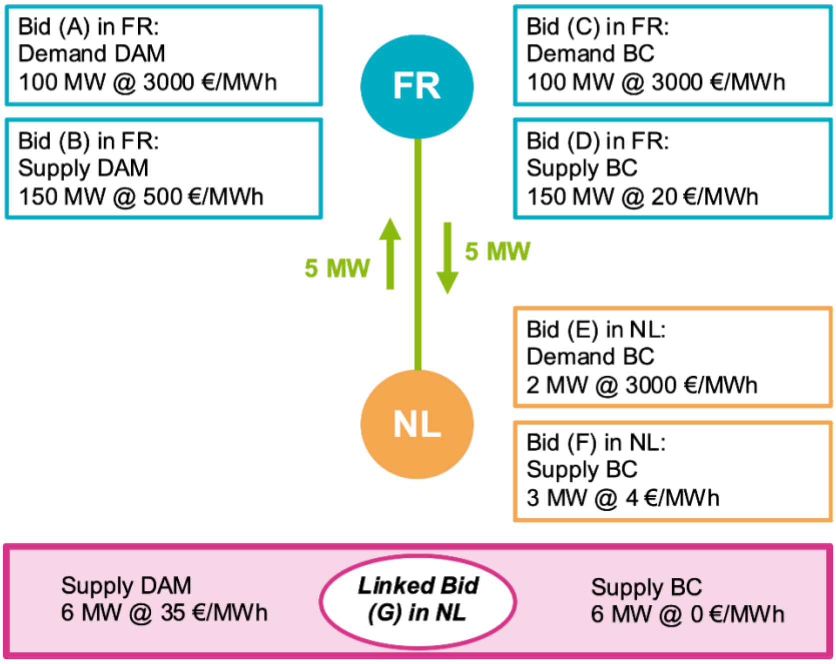

Figure 18: Illustrative example used for highlighting results differences between scenarios

Table 10 describes (i) the cross-zonal capacity allocated to each market (white), (ii) the amount of energy and balancing capacity accepted for each bid (orange), and (iii) the obtained price for each market in each bidding zone (green). These data are provided for every aforementioned design option.

Scenarios	One-Step Multilateral (A1)	One-Step Unilateral (A2)	Two-Step Unilateral (B2)
CZC Split (NL → FR)	DAM: 5 MW BC: 0 MW	DAM: 0 MW BC: 5 MW	DAM: 0 MW BC: 5 MW
(A) Demand DAM in FR	100 MW	100 MW	100 MW
(B) Supply DAM in FR	95 MW	100 MW	100 MW
I Demand BC in FR	100 MW	100 MW	100 MW
(D) Supply BC in FR	100 MW	95 MW	95 MW
(E) Demand BC in NL	2 MW	2 MW	2 MW
(F) Supply BC in NL	1 MW	1 MW	1 MW
(G) Linked DAM in NL	5 MW	0 MW	0 MW
(G) Linked BC in NL	1 MW	6 MW	6 MW
DAM Price FR	500 €/MWh	500 €/MWh	500 €/MWh
BC Price FR	20 €/MWh	20 €/MWh	20 €/MWh
DAM Price NL	39 €/MWh	484 €/MWh	[0…500] €/MWh
BC Price NL	4 €/MWh	4 €/MWh	4 €/MWh

Total Welfare	556321€	554096 €	554096 €

Table 10: Results of Example 4.1

From Table 10, it can be observed that the results from Scenario A2 and B2 are the same in terms of bid acceptance. These results are not the same as for Scenario A1 which lead to inefficiencies in terms of total observed welfare.

Moreover, as it is shown by the previous table, although all the volumes obtained under A2 and B2 are the same, there is an imperfect price equilibrium in the 2-step case B2. Indeed, under scenario A2, the economic links between balancing capacity, day-ahead clearing prices and cross-zonal capacity value are naturally present. In particular, the CZC value is identical for the two markets:

- CZC value for balancing capacity = 20€-4€= 16 €
- CZC value for energy = 500€ - 484€ = 16€.

These are coherent values: CZC is fully allocated to BC; while there is no economic gain possible when allocating more CZC to energy (i.e. optimality condition). This equilibrium CZC price is determined during the single optimization step by the dual variables and KKT conditions (see discussions in Appendix C).

However, if energy is cleared in a distinct subsequent step, such an optimality condition is no longer naturally enforced. Indeed, under B2, any DAM price between 0 and 500 €/MWh can be chosen for day-ahead in NL (as any price within this range is compatible with the accepted volumes). For the moment, in the current configuration of EUPHEMIA, the mid-point price will be return (250 €/MWh). This then implies that the CZC value for energy = 500€ - 250€ = 250€. As the CZC value for balancing capacity is still 16€, the CZC split doesn't appear as optimal: given that the CZC value has higher value for energy than for balancing capacity, there are (in principle) possible economic gains when allocating more CZC to energy (i.e. optimality conditions are not met). We refer to this phenomenon under the term "volume coupling effect" in reference to the price discrepancies that occur in other sequential coupling schemes such as ITVC/EMCC (interim tight volume coupling between CWE and Nordic region, as implemented prior to NWE) or MRLVC (Multi-regional loose volume coupling between Great-Britain and EU, as considered for the implicit coupling of GB and EU following Brexit).

NB: For Scenario A1, the CZC value for balancing capacity is also 16€, while the CZC value for energy is 461€ (=500-39). This remains a coherent result since all CZC has – for this scenario – been allocated to energy, i.e., to the most valuable market.

Example 4.2: Rejection of in the money DAM bids with unilateral linking one-step

A slightly modified version of the previous example is presented in Figure 19 and will be the basis of the following analysis. Note that the changes compared to the previous version are highlighted in red. In this version, on top of the linked bid proposed in NL for the day-ahead market, there is also an expensive supply energy bid.

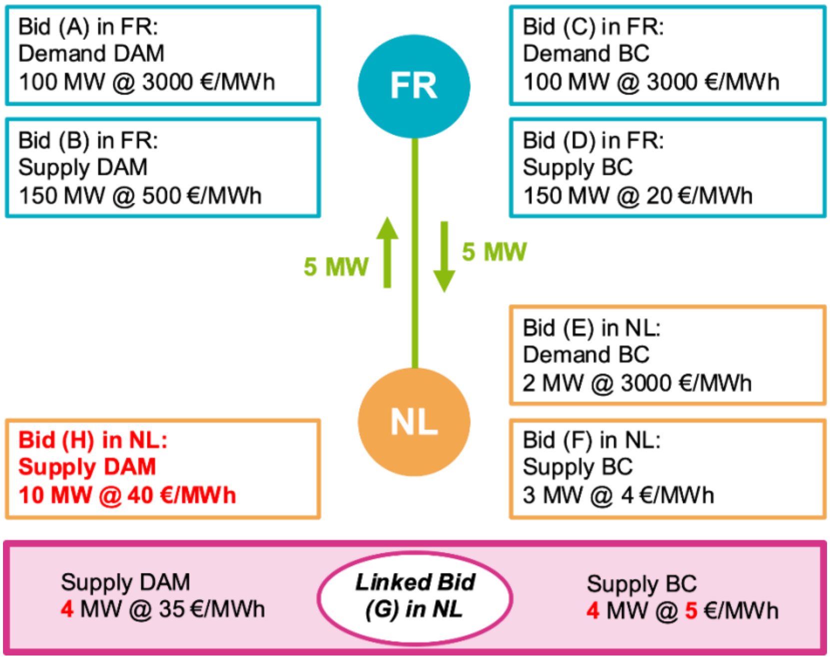

Figure 19: Illustrative example used for highlighting results differences between scenarios

Table 11 presents the outcomes of the three scenarios in terms of bid acceptance, cross-zonal capacity split between day-ahead and balancing capacity and finally the different prices for each market applied in each bidding zone.

From these simulations, we can observe that A1 and B2 provide the same (optimal) results. Scenario A2, the linked bid is entirely rejected, although it is in the money. Let us understand why. During the "BC Dispatch attempt" calculation, an allocation of 100% of the linked bid G to balancing capacity is seen as plausible: given the absence of energy bids (as is the case in this calculation step), BC bid G is cleared as it outbids D to satisfy C. Therefore, its acceptance ratio is fixed to 100% while removing the links for the subsequent calculation. Though, during the actual matching, energy bid H outbids B, and the value created by exporting H to FR on the energy side is higher than the value created exporting G to FR on the balancing capacity side. Hence, the CZC is allocated to energy, and bid G is not cleared on the balancing side (while it cannot be cleared on the energy side because this has been forbidden by the "BC dispatch attempt"). As a result, bid G is not cleared at all.

The situation is different for Scenario B2: while the output of the first step is the same as for Scenario A2, the calculation of the second step re-enables the possibility to match bid G in energy since it is now sure that it is not cleared in the balancing market (and that therefore the constraint over its acceptance ratio can be released). With such a setup, bid G outbids H and is cleared in the final results.

Scenarios	One-Step Multilateral (A1)	One-Step Unilateral (A2)	Two-Step Unilateral (B2)
CZC Split (NL → FR)	DAM: 5 MW BC: 0 MW	DAM: 5 MW BC: 0 MW	DAM: 5 MW BC: 0 MW
(A) Demand DAM in FR	100 MW	100 MW	100 MW
(B) Supply DAM in FR	95 MW	95 MW	95 MW
(C) Demand BC in FR	100 MW	100 MW	100 MW
(E) Supply BC in FR	100 MW	100 MW	100 MW
(E) Demand BC in NL	2 MW	2 MW	2 MW
(F) Supply BC in NL	2 MW	2 MW	2 MW
(G) Linked DAM in NL	4 MW	0 MW	4 MW
(G) Linked BC in NL	0 MW	0 MW	0 MW
(H) Supply DAM in NL	1 MW	5 MW	1 MW
DAM Price FR	500 €/MWh	500 €/MWh	500 €/MWh
BC Price FR	20 €/MWh	20 €/MWh	20 €/MWh
DAM Price NL	40 €/MWh	40 €/MWh	40 €/MWh
BC Price NL	4 €/MWh	4 €/MWh	4 €/MWh

Table 11: Results of Example 4.2

Example 4.3: Infeasibilities in 2-step scenarios

This example can be applied to any linking scheme since it does not account for any links. It aims at showing the infeasibilities that can arise due to the simplification in the CZC split step of the DAM market in a two-step scenario. Let's consider the setup presented in Figure 20.

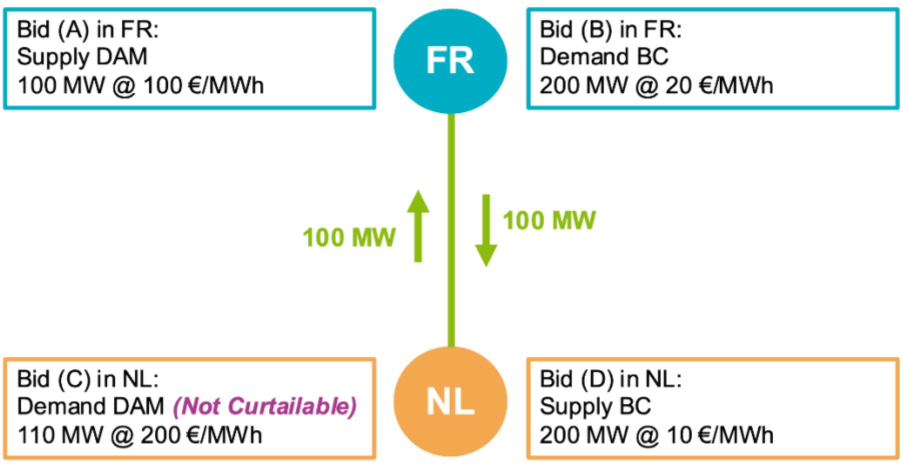

Figure 20: Illustrative example used for highlighting infeasibilities arising from the simplification of the representation of the DA market for the CZC allocation

In the first step of a two-step scenario, the CZC split is computed by relaxing the non curtailability constraint of bid C. The output of this computation will accept every bid fully except bid C for which only 100 MW will be accepted. The outcome in terms of flow will be that 200 MW in the direction NL to FR will be allocated to BC while a cross zonal split of -100 MW will be allocated for DAM. Therefore, in a second step, the DAM flow will be forced to be 100 MW going from FR to NL. Such an "enforcement" is implemented via negative ATCs in the second step. In this case, the ATCs for energy are: ATC_FRtoNL = 100MW; ATC_NLtoFR = -100MW. But this will not be possible because bid C must now be considered as an indivisible bid. Therefore, this problem is infeasible.

Summary

With these three illustrative examples, we show the shortcomings of using unilateral linking in a 1-step setup but also of using a 2-step design.

First, it is shown that the last design (Scenario B2) can lead to imperfect price equilibrium since only a limited set of information is passed by the algorithm to the second step of the process (i.e., volume coupling effects).

Secondly, the next example demonstrates how one-step unilateral linking (Scenario A2) can reject day-ahead bids that are price-compatible/in the money and that would have been accepted in a multilateral setup.

Finally, the last example shows that the separation in a two-step process can lead to the infeasibility of the second step (i.e. infeasible clearing of the day-ahead market).

5. Welfare maximization and procurement cost minimization

In the Single Day-ahead Market Coupling (SDAC), the criterion underlying market operations is the maximization of the welfare. In a context of inelastic demand, as envisioned for the procurement of balancing capacity, and in a pay-as-bid scheme, welfare maximization is equivalent to procurement cost minimization: this design choice is called bid-cost minimization, e.g., in [21,22]. Due to challenges associated to another design options called (marginal-price) "procurement cost minimization" discussed below, and in view of the constrained timeline for the present co-optimization study, this welfare maximization criterion has been retained for the co-optimization prototypes and simulations described in Section 6.

We consider below the possibility of an alternative market design: procurement cost minimization, as envisioned by ENTSO-E in the IIA [4]. This alternative approach seeks to minimize the total procurement costs of balancing capacity based on "marginal-pricing", i.e., where the procurement costs are calculated in a situation where all BSPs in the same market (location and Market Time Unit) are paid the same marginal price.

It is an interesting market design choice, which was already discussed in the IIA [4], and which has been discussed in past literature in the context of US market design, and was specifically advocated by Southern California Edison in the past [20]. It also received attention in the literature [25, 26, 27, 21, 28, 22, 29, 30], though to the best of our knowledge, none of the existing electricity markets worldwide implementing co-optimization have favored this approach. We also did not identify relevant literature that discusses “hybrid objective functions”, as we presume that – despite the discussion held in this section – energy day-ahead market clearing will remain based on the “total (energy) welfare” objective function.

Minimization of marginal price procurement cost is by construction computationally highly challenging and a priori non-scalable (hence intractable for large-scale instances) because it introduces multiplications of prices and quantities in the objective function, thereby yielding bilinear non-convex terms, or in a context of inelastic demand, may otherwise at least require a large number of binary variables to model cost-recovery conditions of accepted balancing capacity bids. We will also see below that prices supporting a market equilibrium (no paradoxically accepted/rejected bids and coherent CZC allocations in view of the zonal prices) may also not exist for solutions minimizing marginal-price (procurement) costs.

Moreover, it may induce adverse bidding behavior, as pointed out in [21] and further studied in [22]. Also, a preliminary study in [16] shows that, with strategic bidding behavior, both auction types (welfare maximization or procurement cost minimization) are subject to efficiency losses, however payment cost minimization is more likely to incur a larger efficiency loss (although the difference is generally small with respect to the reduction in consumer payment).

We review below in Section 5.1 the fundamental competitive equilibrium properties of welfare maximizing solutions (i.e., bid-cost minimization solutions). Section 5.2 then presents two examples illustrating the difference between welfare maximization and procurement cost minimization (also called marginal-price cost minimization): a single bidding zone example, and a second example involving a cross-zonal capacity allocation, notably in terms of equilibrium properties that can be ensured in each case.

Section 5.3 concludes by describing the approach implemented in the Euphemia prototypes developed in the frame of this study, summarizing the computational and design challenges of procurement cost minimization, and proposing next steps.

5.1. Welfare maximization and competitive equilibria: no paradoxical market results

The objective of the Single Day-Ahead Coupling market is to maximize the total welfare of market parties. A main objective pursued when maximizing welfare is to achieve efficient outcomes whereby the matching of orders is the best possible in terms of total utility of the matched demand minus the total supply costs to meet that demand. This welfare is measured based on *declared* utilities and costs, and hence approximates the welfare actually resulting from the exchanges taking place in SDAC.

Crucially, welfare maximization also achieves a second objective: the determination of a so-called competitive equilibrium[11], where market prices are such that there are no paradoxically accepted or paradoxically rejected orders (also called in this study paradoxically allocated orders when linked bids are in scope), and the CZC is optimally allocated in view of the locational prices (ensuring the coherence of the CZC allocation and its marginal value related to price spreads across locations). This second objective is rooted in a fundamental equivalence (that can be mathematically proven) between welfare maximizing solutions and solutions for which one can find prices supporting a competitive equilibrium, with the appealing properties described above (no paradoxical acceptance or rejection of orders, and the CZC optimally allocated given the locational/zonal prices), see for example [31, 32].

This mathematical equivalence is linked to important duality concepts in mathematical optimization, which can be seen for example through the lense of so-called primal, dual and complementarity conditions (or more generally KKT conditions) as optimality conditions of convex optimization problems[12]. The duality concept is illustrated in Appendix C of the present report, and is interpreted in economic terms for the case of co-optimization through a simplified example.

This equivalence between solutions supporting a market equilibrium and market clearing solutions achieving the market objective of maximizing welfare, is summarized in Figure 21 which also highlights differences with the procurement cost minimization setup.

[11] We consider here a standard well-behaved "convex market" setup, leaving aside the additional considerations required when indivisible bids and other sources of non-convexities are in scope. Essentially, in the current SDAC design, the only allowed deviation from a competitive equilibrium is to tolerate paradoxically rejected indivisible bids (so-called block orders): equilibrium requirements are still enforced for "curve orders" and networks.

[12] Alternatively, the fundamental equivalence concept can easily be shown as a direct consequence of what is called "(strong) Lagrangian duality" for linear or convex optimization problems, see e.g., Appendix A in [33].

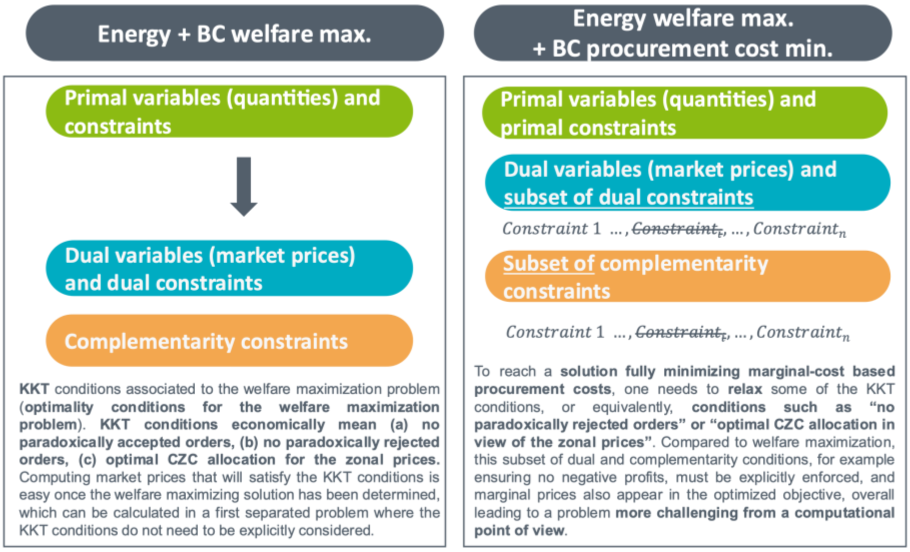

Figure 21: Welfare maximization versus procurement cost minimization and market equilibria.

These differences in terms of reachable market equilibria are illustrated in the next section.

5.2. Two examples on welfare maximization versus procurement cost minimization

We consider here two examples illustrating the differences in terms or market equilibrium properties between welfare maximization and procurement cost minimization:

- Example 5.1: a single bidding zone example originally provided by ENTSO-E during the project, illustrating the differences in the outcomes and the fact that the procurement cost minimization leads to paradoxically rejected orders.
- Example 5.2: a second example involving a CZC allocation, illustrating that CZC may not be optimally allocated in view of the zonal market prices when procurement cost minimization is in scope.

Example 5.1 with a Single Bidding Zone and multiple BC products

During the course of the study, ENTSO-E stakeholders have provided an interesting example that illustrates that (marginal-price) procurement cost minimization and welfare maximization may lead to different market clearing outcomes. We repeat this example here, and discuss the guiding principles of following a welfare maximization approach.

Consider the orders that are presented in Figure 22. The demand for aFRR is inelastic and equal to 20 MWh, while the demand for mFRR is inelastic and equal to 50 MWh. The supply side of aFRR consists of three bids of 10 MWh each at the prices indicated in the figure, and the supply side of mFRR consists of six bids of 10 MWh each at the prices that are indicated in the figure. Note that bid LB is a Linked Bid offered in both markets, and in fact at different prices: 52 €/MWh in the aFRR market, and 55 €/MWh in the mFRR market. LB cannot be accepted in both markets simultaneously.

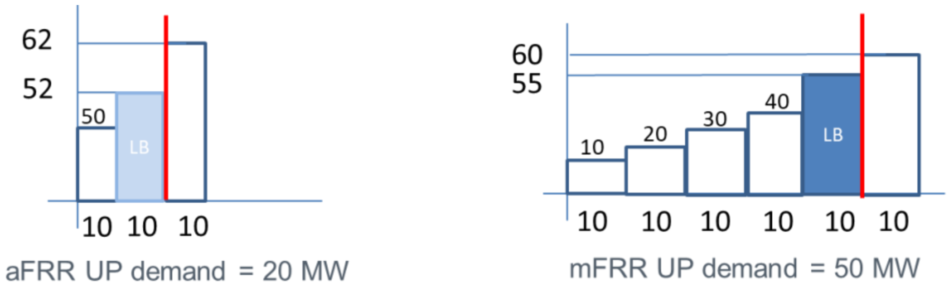

Figure 22: aFRR and mFRR order book in an example that illustrates that payment cost minimization and welfare maximization can produce different market clearing outcomes.

As we will see below, the procurement cost minimizing solution will be to match LB in the mFRR market and compute the following prices:

- price aFRR = 62 (minimum price avoiding negative profits to matched aFRR bids)
- price mFRR = 55 (minimum price avoiding negative profits to matched mFRR bids)

Such a choice leads to a total (marginal-price) procurement cost of balancing capacity of 3990€ (50MW @ 55€/MW and 20MW @ 62€/MW), reported in Table 12.

For these prices, LB would prefer to be cleared in the aFRR market, since it would be remunerated with a profit of 10€/MWh (=62-52), while it generates no profit in mFRR. LB is hence paradoxically allocated in the sense that it is paradoxically rejected in the market where it could have had a better profit[13]. Note that for this matching of bids, it is impossible to find other prices such that there are no paradoxically rejected bids, nor paradoxically accepted bids: the mFRR price must be at most 60 €/MWh in view of the last bid rejected there with this limit price (otherwise this rejected bid will be "paradoxically rejected"), and the price in the aFRR market must be at least 62 €/MWh in view of the last accepted bid there (to avoid negative profits). Hence, prices avoiding to have a paradoxically allocated bid LB matched in the mFRR market do not exist, since the price in the aFRR market must be higher than the price in the mFRR market in view of the other bids and their acceptances.

The phenomenon is related to the fundamental equivalence discussed above in Section 5.1, which does not hold for procurement cost minimization (see Figure 21 for a summary).

Let us now consider the welfare minimizing (i.e., the bid-cost minimizing solution) which can be shown to consist of clearing LB in the aFRR market. The corresponding procurement costs in a pay-as-bid scheme are reported in Table 12. In this case, the market price[14] of aFRR would be at least 52 €/MWh, while the market price of mFRR would be at least 60 €/MWh (to avoid negative profits for accepted bids). One can check that, already, for these minimal prices, the

[13] Note that this more general definition of paradoxical rejection applicable to linked orders, called here paradoxical allocation, does not consider an in-the-money order rejected in a given market as "paradoxically rejected" if that rejection is due to the order being accepted in another market where it is more profitable. Such a definition is used in the current Euphemia implementation for Linked and Exclusive block bids.

[14] Note that this clearing price is not unique, but serves to illustrate the point of the example.

total (marginal-price) procurement costs will be higher than if LB were matched in the mFRR market as above (20 MWh @ 52 €/MWh and 50 MWh @ 60 €MWh, leading to a total of 4040 € reported in Table 12). The intuition of this example is the following: although it is economically efficient to allocate the LB offer in aFRR (because its offered cost is lower in this market, and the next offer in the aFRR merit order is quite expensive), from the point of view of payments it is preferable to match it with the mFRR demand, in order to keep a lower mFRR clearing price *which applies to a larger overall quantity*. More specifically, the possible outcomes are illustrated in Table 12.

On the other hand, for this welfare maximizing solution (matching of bids), if we set market prices respectively at 57 €/MWh in the aFRR market and 60 €/MWh in the mFRR market, no market parties face a paradoxical rejection or paradoxical acceptance: these prices support a competitive equilibrium. The phenomenon is a direct illustration of the mathematical equivalence between welfare maximization and competitive equilibria evoked in Section 5.1 (again, see the summary in Figure 21: a matching of bids and CZC allocation is welfare optimal if, and only if, one can find prices supporting a competitive equilibrium for that allocation (this principle can essentially be traced back to the fundamental contribution of Paul Samuelson in [31]).

Note that, if our goal is to maximize welfare (i.e., minimize procurement costs in a pay-as-bid scheme), then we prefer to match LB in aFRR (highlighted in green in column 2), whereas if our goal is to minimize payments then we prefer to match LB in mFRR (highlighted in green in column 3).

	Welfare (Bid costs)	Procurement costs
LB matched for aFRR	2620 =(50x10+52x10)+(10x10+20x10+30x10+40x10+60x10)	4040 (=52x20+60x50)
LB matched for mFRR	2670 =(50x10+62x10)+(10x10+20x10+30x10+40x10+55x10)	3990 (=62x20+55x50)

Table 12: Welfare and payment outcomes for Example 5.1.

Finally, note that we have the following inequalities, which always hold by design (assuming the same bids are submitted in the two cases)[15]:

$$\begin{aligned}&\mathit{Minimum\ Bid\ Procurement\ Costs}\\ &\quad\leq \mathit{Minimum\ Marginal\ Price\ Procurement\ Costs}\\ &\quad\leq \mathit{Marginal\ Price\ Procurement\ Costs\ of\ the\ Bid\ Costs\ Minimizing\ solution}\end{aligned}$$

Hence, leaving aside the important question of bidding behaviors under the different design options (re-discussed in Section 5.3 below), the design choice leading to the smallest

[15] Here, "Minimum Bid Procurement Costs" denotes what the auctioneer pays when implementing a welfare maximization design combined with a pay-as-bid scheme, "Minimum Marginal Price Procurement Costs", what the auctioneer pays when the auctioneer implements a procurement cost minimization design, and "Marginal Price Procurement Costs of the Bid Costs Minimizing solution", what the auctioneer pays when implementing a welfare maximization design combined with a pay-as-clear scheme.

procurement costs consists of a welfare maximization choice combined with a pay-as-bid pricing scheme. However, bidding behaviors in the case of a minimum procurement cost design may lead to higher procurement costs than in the case of welfare maximization combined with marginal pricing, and the same remark naturally applies to the pay-as-bid scheme variant in the case of welfare maximization.

Example 5.2 with a CZC allocation and a single BC product

The following example is directly inspired from Example 1 above. There are essentially two differences here. First, instead of LB being a multilaterally linked order, it is a single simple order located in a different bidding zone (DE), and the "multilateral link" is replaced by a PTDF constraint limiting DE exports to 10 MW (which to a large extent is similar in nature compared to a bid linking constraint, given that the export constraint forbids LB to be accepted in both markets simultaneously). As LB is now a single simple order, it will have a single limit price equal to 55 €/MWh, whatever the market in which it is finally matched. Second, instead of having demands in balancing markets for two different products, we have two markets for the same product (aFRR) but two different bidding zones: FR and BE. The setup of the example is depicted in Figure 23.

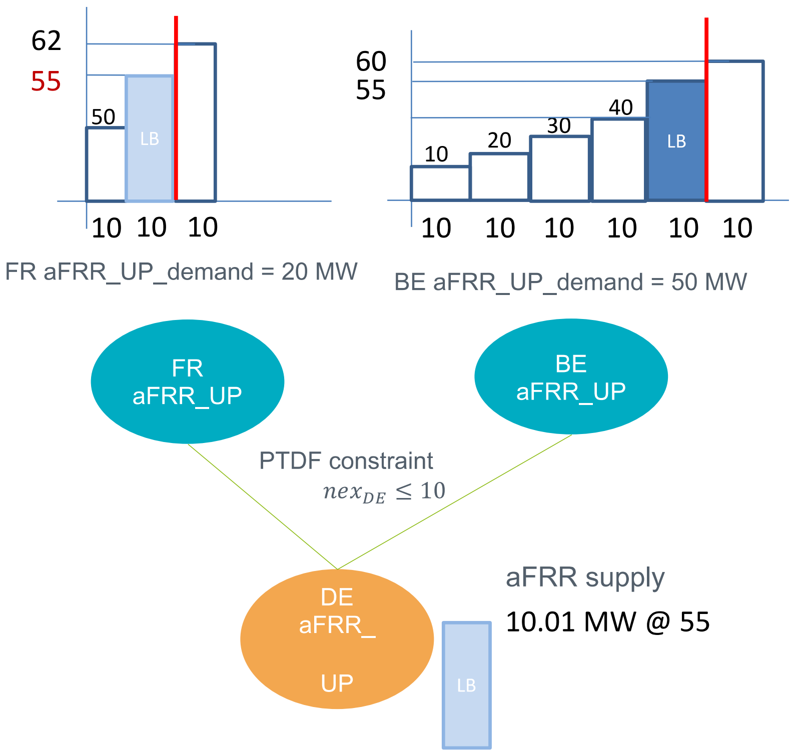

Figure 23: Multi-zone single-product example illustrating differences between welfare maximization and procurement cost minimization (Example 5.2).

One can straightforwardly check that the procurement cost minimizing solution is to match LB in BE, and determine the following zonal market prices for aFRR: FR = 62, BE = 40, DE =

55. In this example with congestion (the CZC is used to full capacity), prices are set in each zone to the limit price of the cheapest BC bid in that zone, the objective being to achieve the smallest marginal-price procurement costs while avoiding negative profits to BSPs. (More generally, the requirement communicated by ENTSO-E is to set market prices to the price of the cheapest BC bid in uncongested areas, here reduced to bidding zones.)

This leads to a sub-optimal allocation of the CZC, since the CZC is used to export aFRR from DE where the price is 55 €/MWh to BE where the price is 40 €/MWh, even leading to a negative congestion rent, while the price in FR is 62 €/MWh. A coherent, optimal CZC allocation for these zonal prices would be to use the CZC to export from DE to FR. To avoid a negative congestion rent, the price in BE could be increased to 55 €/MWh, but this would not change the fact that the CZC is not coherently allocated according to the zonal price differences (it should be allocated to exchanges between DE and FR, not between DE and BE), which in a well-behaved welfare maximization context corresponds to the (marginal) market value of the CZC.

The only way to restore optimality of the CZC allocation would be to set a price in BE equal to the price in FR, i.e., equal to 62 €/MWh, but in that case, the most expensive bid in BE with a price of 60 €/MWh would then become paradoxically rejected. Moreover, such a price system would then be making the matching of bids and the underlying CZC allocation more costly than the one discussed next, resulting from welfare maximization combined with zonal prices supporting a competitive equilibrium (optimal CZC allocation and no bid paradoxically accepted or paradoxically rejected).

We now analyze the results for the welfare maximizing solution, which is to export the bid LB to the FR market instead of the BE market. For this bid matching and underlying CZC allocation, one can determine the following prices, ensuring that no bid is paradoxically accepted/rejected, and that the CZC allocation is optimal given these zonal aFRR prices: Price FR = 60, Price BE = 60, price DE = 55. Indeed, for these zonal prices, the network operator is, in terms of congestion rent maximization, indifferent to exporting from DE to FR, or from DE to BE.

	Welfare	Procurement costs (= TSO BZ payments – Congestion Rent)	Congestion Rent	TSO BZ payments
LB matched for aFRR FR (Welfare max. solution)	2650	4150	50 € (= *10x(60-55)*)	4200
LB matched for aFRR BE (Procurement cost min. solution)	2670	3390 *(62x20 + 40x40 + 55x10)*	–150 (= *10x(40-55)*)	*3240 € (62x20 + 40x50)*

Table 13: Welfare and payment outcomes for Example 5.2.

It is important to emphasize what negative congestion rents imply in the case of procurement cost minimization: concretely, when congestion rents are negative the BSPs are paid more than what is collected from TSO demand bids, which means that the auctioneer of the balancing market (namely, the TSOs) are financially exposed for covering the shortfall. This is guaranteed not to occur in the case where prices are set and orders are allocated according to a welfare maximization criterion.

5.3. Design and computational challenges of the marginal-cost minimization approach

The discussions above have highlighted a key design challenge that emerges in the marginal-cost minimization design, namely that standard market equilibrium conditions, which are synonymous to avoiding paradoxically accepted/rejected orders and ensuring optimal CZC allocations given the zonal prices (in particular avoiding negative congestion rents, but more generally allocating CZC to the market which has the highest value – measured by the cross-zonal price differences), cannot be guaranteed.

In terms of market design challenges, the literature points out gaming risks: as highlighted above, procurement cost minimization may induce adverse bidding behavior, as pointed out in [21] and further studied in [22], and a preliminary study in [16] shows that with strategic bidding behavior, both auctions are subject to efficiency losses, but payment cost minimization is more likely to incur a larger efficiency loss (although the difference is generally small with respect to the reduction in consumer payment).

On the computational front, procurement cost minimization represents a formidable challenge as either multiplications of price and quantity variables should be handled (yielding non-convex quadratic terms), or a huger number of binary variables to model the avoidance of negative profits for BC bids. The huge computational challenge that this design choice represents is for example highlighted in [30].

As a way forward, a dedicated study on the topic can further analyze market design challenges evoked above in collaboration with stakeholders, and could also assess if scalable methods can be proposed to tackle procurement cost minimization.

Part II: Prototyping and Simulations

6. Description of the prototypes and simulation setups

Based on the three scenarios presented in Section 4, three prototypes were created and used to perform simulations in order to assess the technical feasibility and performances of the co-optimization of energy and balancing capacity considering cross zonal available capacity for both usages. The present section is devoted to the presentation of the dataset used to perform the simulations along with a description of the adaptations made in Euphemia to create the three different prototypes based on their corresponding scenario.

6.1 Prototype description

We highlight here the main elements of the adaptations made in Euphemia to perform the simulations. The Euphemia prototype is based on Euphemia 11.1. Euphemia has been modified essentially along two dimensions:

- Network models have been adapted so as to create for each bidding zone of the input topology, a bidding zone for the energy market, and a bidding zone for the balancing capacity market (mFRR up). Moreover, constraints related to model O3 have been implemented to enforce the deterministic requirement (see Section 4).

- Multilaterally linked orders have been introduced as new bidding products.

This prototype has been used as the basis for all simulations.

For Scenario B2 (2-step with unilateral bid linking), the first step is performed with a simplified version of this prototype, where all bid indivisibility requirements are dropped: concretely, so-called block orders are considered as divisible, and essentially a large continuous problem is solved when taking this simplification into account[16].

6.2 Simulation Data

Based on the description of the different scenarios and their implementation as prototypes given above, it can be observed that, in order to perform the envisioned simulations, the following datasets are required:

- Initial cross zonal capacity (CZC) and network coupling related data
- Supply bids and TSO capacity demand for the Balancing capacity market
- Supply and demand bids for the Day-ahead market

[16] Note that in the current implementation, some binary variables remain in the simplified problem to manage so-called Scalable Complex Orders, but their impact is negligible.

- Essence of the link between concerned linked supply balancing capacity and energy bids

Note that these sets of data must cover the European regions that are currently covered by EUPHEMIA to assess the feasibility and tractability of co-optimization at a European level. Furthermore, the equivalent of 3 months of daily simulations were run to evaluate the feasibility of the presented setups.

Initial cross zonal capacity and network information

For the simulations, we used historical network data of the day-ahead market. In other words, historical capacities on ATC lines and historical PTDF constraints were employed.

Some network features were however dropped:

1. Ramping constraints on lines, lines sets and zonal net positions. Ramping features are not incompatible with co-optimization of energy and balancing capacity. Nonetheless, supporting them in our test prototypes would have significantly increase the development work due to additional modifications to be made to the mathematical model of EUPHEMIA. These features were thus dropped for practical reasons but could theoretically be kept.
2. Capacity constraints on lines sets. Again, supporting capacity constraints on lines sets together with co-optimization of energy and balancing capacity is possible but was deemed beyond the scope of this assignment due to the additional modifications needed in the mathematical model of EUPHEMIA.
3. LTA inclusion. This means the historical virgin flow-based domains were used as final flow-based domains.

Some existing network features were kept while their relevancy might be questionable in a co-optimized framework:

1. Losses on lines. It might be wise to reflect upon the relevancy of the current mathematical model when the energy and the balancing flows have opposite directions on a lossy line.

Balancing Capacity supply bids and TSO demand

First, note that the current simulation scope only considers a single balancing capacity product, namely mFRR up. The extension of the market design analysis regarding the co-optimization of energy and balancing capacity along with the simulation results provided in this document are not in the framework of the current study. Therefore, the impact on performances and market design properties of the co-existence of different balancing products such as for example up and down or aFRR, mFRR and RR is important to be investigated in the future (see Section 8.1 on remaining high-level design open questions).

Several TSOs provided data regarding balancing capacity supply bids in their region for several days. For these regions (Belgium, Netherlands, Germany, Slovenia, Croatia, Hungary and Spain), the data provided were used for 4 representative days and these 4 days were randomly repeated to form a 3-month dataset.

For the countries that did not provide any bids, the supply bids of Belgium were used with some adjustments. The power offered by each supply bid was scaled by the DA cleared volume factor between the two countries. This factor is computed as the total day-ahead energy volume cleared in that country in February 2022 divided by the one cleared in Belgium at the same time. With respect to supply bid prices, a small additive gaussian noise was applicated to avoid replicating balancing capacity market prices between countries.

Regarding TSO demand in each bidding zone for countries that did not provide this information, it was assumed to be equal to 90% of the total offered balancing capacity in that bidding zone. Regarding its bidding price, a price of 3000 €/MWh was considered which corresponds to the price cap of the day-ahead market. It is equivalent to consider this balancing capacity demand as inelastic.

Links between supply balancing capacity and energy bids

In order to control the proportion of balancing capacity linked supply bids given to the prototypes, only a pre-defined percentage of the created balancing capacity supply bids were linked to energy bids according to the following procedure. First, these balancing capacity bids were randomly selected for each bidding zone among the total set of balancing capacity supply bids. Then, for each chosen linked balancing capacity supply bids, an energy supply bid with the same offered power and price was created and linked to this bid. The link is either multilateral or unilateral depending on the employed prototype.

Day-ahead supply and demand bids

All energy bids (supply and demand) were taken as is from the historical day-ahead data (i.e., no changes were applied) except for the Classical Complex Orders in Spain, Portugal and Ireland that were converted to Scalable Complex Orders. This conversion is usually applied when simulating future market conditions as the Scalable Complex Orders are expected to replace the Classical Complex Orders on the long-term.

Furthermore, due to links between balancing capacity and energy offers, some energy supply bids were created. Therefore, the total set of energy supply bids in each bidding zone is

composed of these created linked energy supply bids and the original set of energy supply bids mentioned above.

7. Simulation Results

In this section, we present results obtained from the simulations run for the different scenarios described in Section 4. We focus on the observed performance of the different prototypes and analyze relations between cross-zonal capacity allocations and zonal price differences in the energy and balancing capacity markets. Market impacts and welfare comparisons are not considered in view of the lack of representative bid data for this purpose at this stage.

7.1 Results for Scenario A1: 1-step multilateral linking

Times to First Solution

The Time to First Solution (TTFS), i.e., the time needed by the algorithm to find a first (high-quality) market clearing solution is a major KPI regarding scalability. Figure 24 presents the Times to First Solution for the first studied scenario.

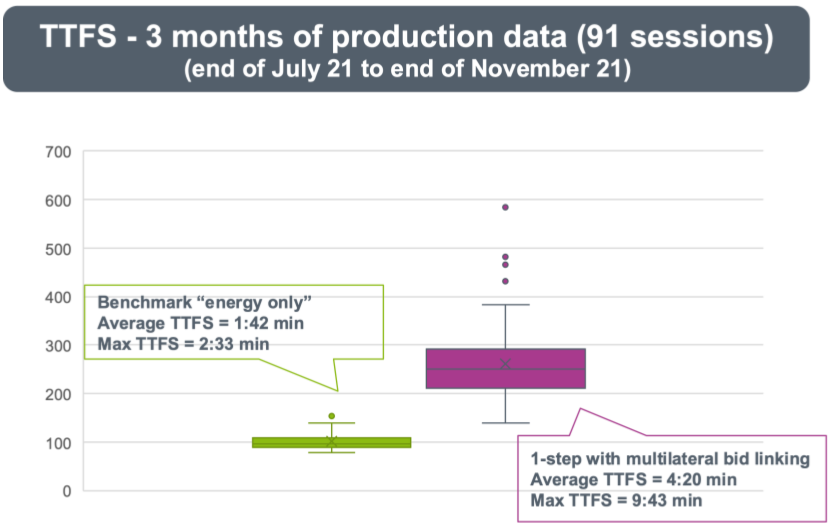

Figure 24: Times to First Solution for Scenario A1 (1-step with multilateral bid linking)

The results show that co-optimization of energy and one balancing product (mFRR up) can be efficiently performed in SDAC when 60' data is in scope[17]. Scaling up to multiple balancing capacity products should be computationally tractable in a 60' context. However, several market and algorithmic design points should still be tackled, including an analysis of the prototype performances in a 15' setup.

Analysis of CZC allocations and zonal price differences

[17] Note that the TTFS is multiplied by about 3.5 compared to an energy-only clearing, mainly due to the number of bidding zones and network model elements multiplied by 2 in order to account for the balancing capacity market.

Based on the results obtained using a sample cross-zonal line and business day presented in the figure below, the price formation along with the relations between cross-zonal capacity allocations and zonal price differences are analyzed in the following paragraphs.

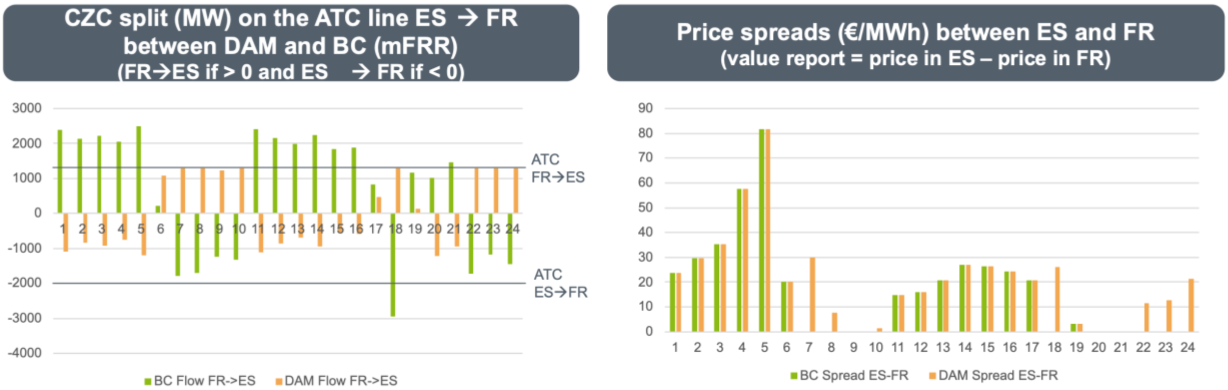

Figure 25: Cross-zonal capacity allocations and zonal price differences for a representative business day (July 26th, 2021) – Scenario A1 (1-step with multilateral bid linking)

These sample test results are rich in terms of observations. Note that other optimal CZC split patterns may exist in practice on top of the ones encountered in this example. A detailed list of the possible CZC split patterns is presented in Appendix C. Furthermore, notice that the empirical observations highlighted in the sequel of this section confirm properties that follow from the theoretical analysis of market equilibrium (presented in detail in Appendix C).

Congested unidirectional CZC split: periods 6, 17, 19 show basic cases where the CZC is used both for energy and balancing capacity in the same direction (FR→ES). This situation can only happen if the zonal price spreads (equal to marginal value of the CZC on the border) are equal in both the energy and balancing capacity markets which is indeed the case during these periods.

Congested CZC split to energy and BC convergence: periods 7, 8, 10, 18, 22, 23, 24 show cases with (i) a positive energy price spread in the direction FR to ES, alongside (ii) all the CZC being allocated to energy in that direction, together with (iii) no BC price spread between the two zones, along with (iv) a BC flow in the direction ES to FR without congestion in that direction.

Uncongested CZC split: periods 9, 20 and 21 show occurrences without zonal price spreads, in which CZC is only partially used, and in different directions for the energy and the balancing capacity markets.

Congested unidirectional CZC split to BC with adverse energy flows: finally, periods 1 to 5 and 11 to 16 reveal that, as allowed by the implementation of co-optimization with ATC lines, a cross-zonal energy flow from ES to FR can free up CZC for the exchange of balancing capacity in the direction FR to ES, as long as this is generating welfare. This "flow netting", as emphasized in Section 1.6, is possible as long as losses (if any) resulting from the flow on the energy side are compensated by gains from the flow on the balancing capacity side. Note that the reverse case in which BC flows would free up CZC for energy is *not* allowed by the models, as the CZC reservation for BC may be different from its usage in real time ("optionality" of BC).

7.2 Results for Scenario A2: 1-step unilateral linking

Times to First Solution

Figure 26 compares the Times to First Solution for this Scenario A2, 1-step with unilateral bid linking, with the ones obtained with Scenario A1.

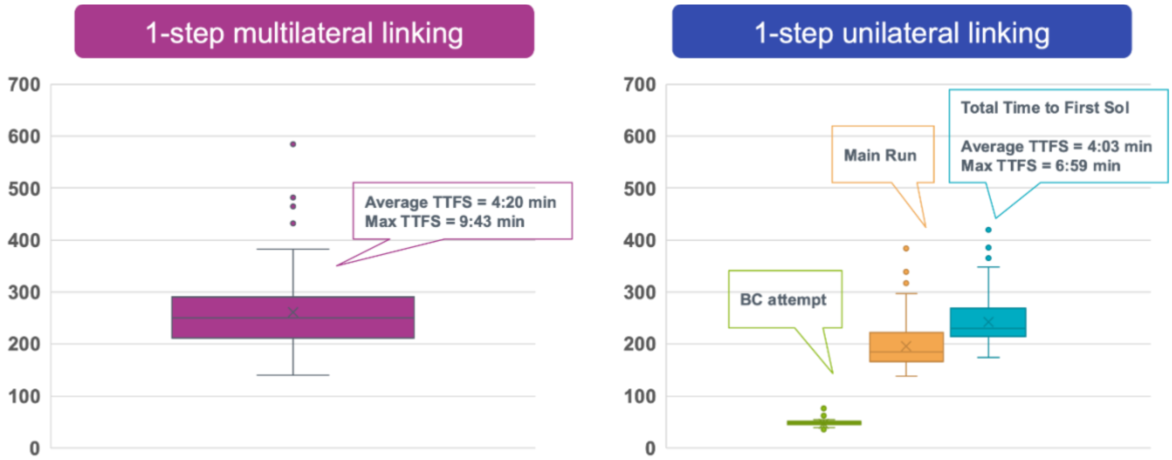

Figure 26: Times to First Solution for Scenario A2 (1-step with unilateral bid linking)

Compared to Scenario A1, 1-step multilateral bid linking, this scenario is slightly faster. The minor improvement of performance is mainly due to "links", coming from bid linking in the price problems in Scenario A1, which are not present in this case. Implementation of bid linking and its impact on price problems in Scenario A1 can however a priori be refined to improve performances via implementation changes in price problems. It can be expected that after such changes, for the simulations in scope, performances with Scenario A1 (1-step with multilateral bid linking) and with Scenario A2 (1-step with unilateral bid lining) will essentially be the same.

Regarding CZC allocations and zonal price differences, observations should be similar to those examined in the results of Scenario A1 as the only difference between the two scenarios concerns the type of bid linking. We will see in the next section that these observations about the coherence of the CZC allocations with respect to zonal price differences do not hold anymore in a 2-step setup.

7.3 Results for Scenario B2: 2-step unilateral linking

Times to First Solution

Figure 27 presents the Times to First Solution for Scenario B2, 2-step with unilateral bid linking. This figure allows us to further compare the results obtained for this scenario with the two others (Scenarios A1 & A2).

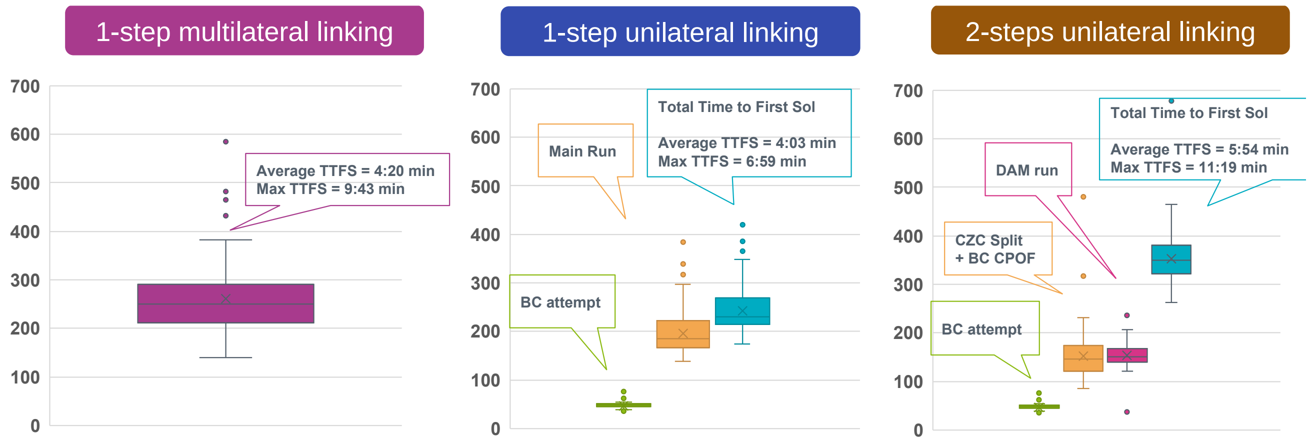

Figure 27: Times to First Solution for Scenario B2 (2-step with unilateral bid linking)

In Scenario B2 (2-step with unilateral bid linking), we can observe that the first step performing the CZC allocation and the BC CPOF is slower than the second step (DAM clearing). This difference is mainly due to having more orders, more bidding zones and more network constraints in the first step since they are needed to perform the CZC split. Overall, this is the scenario leading to the lowest performance since more computations need to be performed. Note that, as mentioned in Section 6.2, the simulations are only performed considering a single balancing capacity product. Further market design analysis and simulations will be needed (as emphasized in Section 8.1) to determine the impact of a larger set of balancing products on the tradeoffs and performance profiles computed and displayed in this study.

Analysis of CZC allocations and zonal price differences

Figure 28: Cross-zonal capacity allocations and zonal price differences for a representative business day (July 26th, 2021) – Scenario B2 (2-step with unilateral bid linking)

From the results presented in Figure 28, we can highlight the following observations.

Suboptimal CZC split detrimental to BC: periods 7, 10, 18 show that in this 2-step setup, a larger share of the CZC can be allocated to energy even though the zonal price spreads are more interesting on the side of the balancing capacity market. Period 1 also shows a case where, in view of the zonal price spread higher in the BC market, an energy flow (from ES to FR) freeing up cross-zonal capacity in the opposite direction (from FR to ES) for the exchange of balancing capacity should have occurred: the value destructed by this energy flow would be compensated by a higher value created by the BC flow.

Suboptimal CZC detrimental to energy: period 21 shows that all the CZC can be allocated to the BC markets even in situations where the zonal price spreads are strictly more interesting in the energy market.

Optimal CZC split: finally, we observe that CZC allocations with respect to zonal price spreads in both markets can still be coherent during some periods, for example at period 2 in which the CZC is fully allocated to the BC market and where the zonal price spreads are equal.

Infeasible second steps

Though the theoretical infeasibilities illustrated in Section 4 above (see Example 4.3) have not been observed in practice in the simulations, operational infeasibilities of the second step of this 2-step scenario have been encountered. Concretely, the cross-zonal capacity given to the energy market clearing problem (i.e., the second step of that scenario) may make that second step infeasible. It is due to the corresponding remaining available margins (RAM) of the flow-based constraints given to the second step which may be slightly too low for the problem to be mathematically feasible, due to the rounding or numerical precision of these RAMs. Such types of operational issues could be overcome by adequate specifications of the rounding process applicable to remaining cross-zonal capacities (e.g., remaining available margins of flow-based constraints) transferred to the second step. However, the risks of infeasibilities highlighted by the concrete Example 4.3 in Section 4 are inherent to the design of the overall 2-step process and cannot be avoided.

7.4 Conclusions of the simulation results

The simulations validate the proof-of-concept implementations of the scenarios in scope. Furthermore, they also show that the 1-step scenarios, compared to the 2-step scenario, avoid incoherent cross-zonal capacity allocations with respect to zonal price spreads, risks of infeasible second steps, and are also faster overall. Additional simulations are required in order to analyze the impact of increasing the complexity of the prototypes by considering additional features such as 15’ Market Time Units, multiple balancing capacity products, and so on.

Note however that the 2-step approach can be seen as a specific heuristic method for computing a market clearing solution, even though key properties in terms of price coherence cannot be enforced in this context (due to effects like volume coupling). In simulation setups where the proposed 2-step design would lead to overall faster calculations than the 1-step calculations as they are currently operated (if the calculation simplifications in step 1 compensate sufficiently the fact that there are 2 steps instead of one), the scalability of the 1-step setup could be further improved by the design of specific heuristics.

Part III: Roadmap

8. Current status and next steps towards an actual implementation

The present study has provided a proof-of-concept implementation of the co-optimization of energy and balancing capacity in a specific restricted SDAC context: 60’ Market Time Units, a single balancing capacity product (mFRR up) and specific types of bid linking constraints, namely mutually exclusive bids including a variant with prioritization of balancing capacity products via a process based on sequential market clearing[18]. As a major achievement paving the way to an efficient co-optimization process, and applicable to any cross-border balancing capacity auction in general, the study has also essentially solved the main challenges attached to the enforcement of the deterministic requirement, as discussed in Section 3[19]. The present investigation has also compared specific market design scenarios by reporting advantages and drawbacks of using (i) multilateral or unilateral bid linking (see Section 2), (ii) 1-step versus 2-step designs as defined in Section 4, or (iii) (marginal-price) procurement cost minimization (see Section 5).

Nonetheless, major as well as secondary market design elements still remain to be clarified before reaching the final specifications of an actual implementation. The main remaining open design questions are discussed in the two following sections below.

8.1 High-level design open points

Bid linking in the presence of multiple balancing capacity products

The present study has only considered a limited set of bid linking constraints by focusing on mutually exclusive bids, either via “multilateral bid linking”, or its “unilateral bid linking” counterpart where a procedure seeks to give priority to specific products. If the prioritization idea underlying “unilateral bid linking” is pursued, a particular open question will be to provide a full set of specifications for this type of link when multiple balancing capacity products are in scope. There is no such ambiguity in multilateral bid linking, which guarantees a sound market design based on well-defined economic first principles pertaining to multi-product auctioning that have already been successfully implemented in other advanced international electricity markets.

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[18] Recall that the first step of the sequential process specified for unilateral bid linking, called “BC Attempt Dispatch”, only sets upper bounds on bid acceptances for the actual clearing in the next step. In this regard, the process can be described as a semi-sequential market clearing process.

[19] Note that further refinements are not excluded in the future in order to further address for example the conservativeness of the retained approach even if its conservativeness was already improved compared to the other proposed tractable approaches.

Beyond mutually exclusive bid conditions, more advanced types of bid linking could be favored by market parties: for example, linking enabling to condition the acceptance of a downward balancing capacity bid to the acceptance of a supply energy bid. SDAC already features linked families of block orders, with so-called “parent-child links” enforcing that a block can be accepted only of its “parent block” is accepted, and provided that certain no-loss conditions are satisfied (generalizing to families of block orders the no-loss conditions applicable to regular block orders). Such conditional acceptances can accordingly be defined for divisible balancing capacity products.

Incoherent prices due to sequential calculations

As observed in Section 1.2, Section 2 and Section 4, sequential market clearing can lead to incoherent prices with respect to the bid matchings. Indeed, even though acceptances of BC bids are not final in the so-called “BC Attempt Dispatch” (seeking to prioritize the BC products over energy products part of linked bids by setting bounds on acceptances), its impact on the final outcomes is such that paradoxically rejected linked bids cannot be avoided. This issue is not present with “multilateral bid linking”. If unilateral bid linking continues to be contemplated, an extensive analysis of its impact on market parties, including on induced bidding behaviors (see Section 1.3 and Section 2 regarding opportunity cost discussions) as well as on risks of price reversals (see below), is strongly recommended.

Similarly, as highlighted in Section 4 presenting 1-step versus 2-step scenarios and demonstrated in the simulations, cross-zonal capacity allocations can be inconsistent with respect to balancing capacity and energy zonal price spreads with a 2-step design. If such a design continues to be considered, an extensive analysis of its impact on TSO processes is recommended by analyzing, for example, its effects on congestion revenue computations, compatibility with LTA inclusion processes, etc.

Risks of price reversals

As highlighted in Section 1, sequential calculations such as those performed with unilateral bid linking and the 2-step approaches can create price reversals: balancing capacity products of higher quality can be priced lower than lower quality products. This topic is further discussed in Appendix A of this report. An investigation of this risk and its dependence on the chosen type of link between balancing capacity and energy bids, together with its implications on bidding behaviors of market participants, is also needed to determine the best market design options.

Welfare maximization versus (marginal-price) procurement cost minimization

Section 5 has provided preliminary insights on employing a market design approach minimizing marginal-price balancing capacity procurement costs. The main challenges of such an approach compared to a welfare maximization design (bid-cost minimization), from a market design perspective, are that (i) procurement cost minimization can lead to cross-zonal capacity allocations which are incoherent with respect to zonal market prices (for example leading to negative congestion rents[20]), and that (ii) bids may become paradoxically rejected. However, if such a design is contemplated in the future, an investigation of its effects on TSO processes and bidding behaviors of market participants would be required. Moreover, an assessment of the computational challenges associated with procurement cost minimization will be required, in order to determine if algorithms, able to tackle large-scale co-optimization instances at the European level, could be proposed.

Flow netting: impact and recommendations

As emphasized in Section 1.6 but also by means of examples and simulations throughout this report, flow netting enables cross-zonal capacity allocated to the energy markets to free up, in some cases, additional CZC for the exchange of balancing capacity. Such situations emerge naturally in flow-based models, where the energy net positions multiplied by their PTDF may lead to an increase of RAM available for the exchange of balancing capacity for particular CNECs. In ATC network models, it is a modelling choice as it is possible to prevent energy flows from freeing up cross-zonal capacity for the exchange of balancing capacity, though this will lead to a sub-optimal usage of the cross-zonal capacity. Further analysis of flow netting effects and their impacts in practice on market results and processes would be beneficial, for example, to identify conditions in which flow netting should be favored or deactivated.

Compatibility with balancing energy market processes

As noted in Section 3, model (O3) is conservation in the sense that it ensures that there also exists an ATC domain within the cross-zonal capacity domain allocated to the balancing capacity markets which satisfies exactly the deterministic requirement. A detailed analysis of the advantage or disadvantage of this features, and of other yet undefined compatibility requirements between the auctioning of balancing capacity in the day-ahead markets in a co-optimization setup with the operations of balancing energy markets, is also needed.

8.2 Detailed design open points

After the presentation of the remaining important high-level design open points, several detailed design points have to be addressed too. These open design points include, for example, the specification of rounding procedures (both for matched quantities, network flows and market prices) bearing in mind the peculiarities of the co-optimization setup and how the risk of infeasibilities in a 2-step scenario, due to rounding, must be addressed if such a 2-step scenario continues to be envisaged. How curtailment should be managed when energy and multiple balancing capacity products are in scope will also have to be carefully specified by either leveraging current existing curtailment management procedures or tailoring them for co-

[20] Negative congestion rents correspond to a financial liability of TSOs who may be required to cover financial gaps in settlements in the BC market. Arrangements are needed to determine how the financing of these gaps is split among TSOs.

optimization of energy and reserve. Finally, note that completing the high-level market design is a prerequisite in order to be able to create an exhaustive list of detailed design open points remaining to address.

9. Timeline

9.1 Timeline

In terms of timeline, the specification of the high-level design is expected to require from 5 months to 8 months, depending on how extensively certain specific design options should be investigated (market designs closer to existing worldwide industry practices would take less time to be fully specified, and are generally exposed to less risk and delays resulting from deviations from economic first principles). The detailed design is expected to require 4 to 5 months after the completion of the high-level design.

In parallel to the high-level and detailed design tracks, a continuous improvement of the existing prototypes will enable to support decision making on the design options with concrete simulation-based quantitative assessments, and to limit efforts required later to implement a production version of the algorithm performing co-optimization.

After the start of the industrialization of the final base design, a period is also foreseen during which last adjustments of the design following from interactions with all stakeholders can be implemented.

In total, from 1.5 years to 2.5 years are expected to be needed to achieve a full implementation of co-optimization in the SDAC algorithm. Nevertheless, the present study already achieves a breakthrough result by introducing co-optimization to the Euphemia algorithm, demonstrating its technical feasibility, and accommodating various extremely challenging novel requirements by TSOs, such as the deterministic requirement.

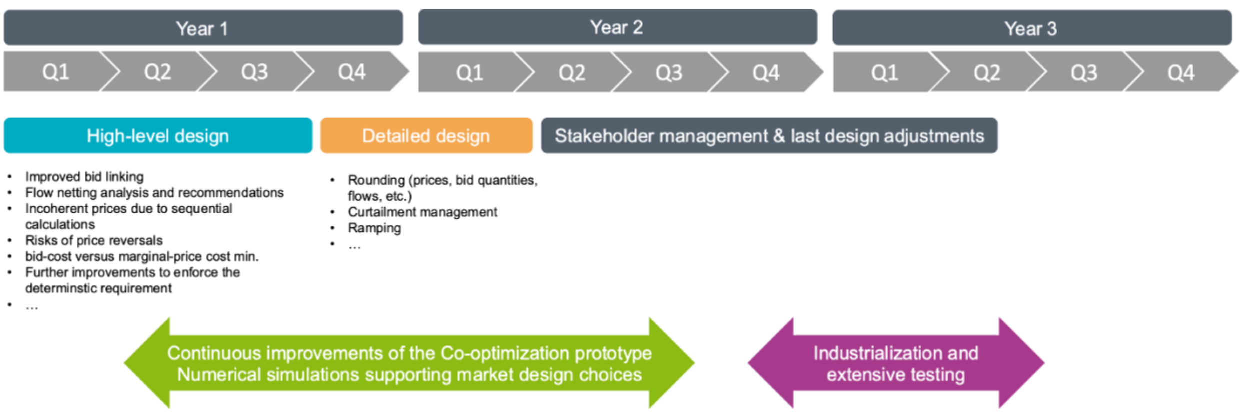


Table 14: Timeline for the implementation of the co-optimization of energy and balancing capacity in SDAC

10. Conclusions

As highlighted in the beginning of this report, the present study on the co-optimization of energy and balancing capacity in SDAC has achieved three main objectives.

Firstly, the study has successfully introduced co-optimization to the Euphemia algorithm by relying on state-of-the-art optimization theory and rapid prototyping, combining deep theoretical expertise within N-SIDE, as well as technological and industry domain expertise that has allowed our team to deliver a viable algorithmic solution forward for pan-European co-optimization. The Euphemia algorithm has specifically been enhanced at prototype scale in order to perform co-optimization of energy and one balancing capacity product (upward mFRR) at the SDAC level. A first quantitative scalability assessment of co-optimization shows that co-optimization can be tractable in a setup with 60' Market Time Units (MTU), though further scalability assessments, together with dedicated performance improvements, are required before being able to implement co-optimization in a 15' MTU setup.

Secondly, the study has clarified key market design concepts considered by stakeholders and enabled to reach a deeper understanding of specific notions and their associated challenges. These concepts include bid linking options, market and pricing implications of the so-called deterministic flow-based compatibility requirement, 1-step versus sequential market operations, welfare maximization in comparison to (marginal-price) procurement cost minimization for the procurement of balancing capacity, flow netting, or risks of price reversals in balancing capacity markets. The study succeeds in identifying an approach to efficiently enforce the deterministic flow-based compatibility requirement which has been validated by multiple numerical experiments. Regarding bid linking, unilateral bid linking has been shown to lead in some situations to paradoxically rejected linked bids (or paradoxically allocated linked bids); a situation that multilateral bid linking is guaranteed to prevent. Furthermore, the qualitative comparison of 1-step versus 2-step scenarios (the 2-step option includes a sequential clearing dimension) has shown that the 2-step design can lead to incoherent cross zonal capacity allocation and zonal prices, or even to infeasible second steps in certain cases. Finally, a first analysis of procurement cost minimization has shown that this design can induce paradoxically rejected bids or incoherence between the cross-zonal capacity allocations and zonal prices differences, including negative congestion rents (which imply a financial liability of TSOs for covering auction financing holes in the day-ahead market).

Given the breadth of the covered topics, open questions or challenges have been pointed out in situations when the efforts had to be restricted to providing preliminary answers on a specific topic.

In conclusion, the study depicts some of the major high-level remaining design points, as well as detailed ones, that still have to be specified before the possibility of an actual implementation of co-optimization in SDAC. To achieve such an implementation in the SDAC algorithm, a roadmap together with a timeline and cost estimates have been proposed in the last sections of the present report.

Appendix A: Price Reversals in Sequential Markets for Reserves

This section describes an example of price reversal in the early California market design which was based on sequential market clearing. The example is inspired by [7], and sourced from [23] (slides 43-50). This issue highlights the importance of **reserve substitutability**, i.e., the property that higher-quality reserve supplies can be used for covering the demand for lower-quality reserves. For instance, a technology that is qualified for the provision of aFRR would typically be considered technologically qualified to provide mFRR *and* RR since its response time is sufficiently rapid. Similarly, a resource which is qualified to provide mFRR can typically be qualified for providing RR.

Substitutability relates to the issue of bid linking but will need to be analyzed in further detail in future studies about co-optimization, since the content in [2] does not address this complex issue adequately.

Example A.1: Sequential optimization can create price reversals

Consider a scenario of sequential optimization whereby the TSO demands 400 MWh of aFRR, and 350 MWh of mFRR. Consider, furthermore, that there are four bids in a sequential market clearing setting:

- Bid 1 offers 600 MWh of aFRR at 10 €/MWh. These bids are also assumed to be eligible for covering the TSO demand for mFRR, because since they are technically qualified for aFRR they are also technically qualitied for mFRR.
- Bid 2 offers 50 MWh of aFRR at 15 €/MWh. These bids are also assumed to be eligible for covering the TSO demand for mFRR, because since they are technically qualified for aFRR they are also technically qualitied for mFRR.
- Bid 3 offers 25 MWh of mFRR at 5 €/MWh.
- Bid 4 offers 400 MWh of mFRR at 20 €/MWh.

The bids are depicted in Figure 29. Red indicates bids that are originally offered in the aFRR market, while green indicates bids that are offered in the mFRR market.

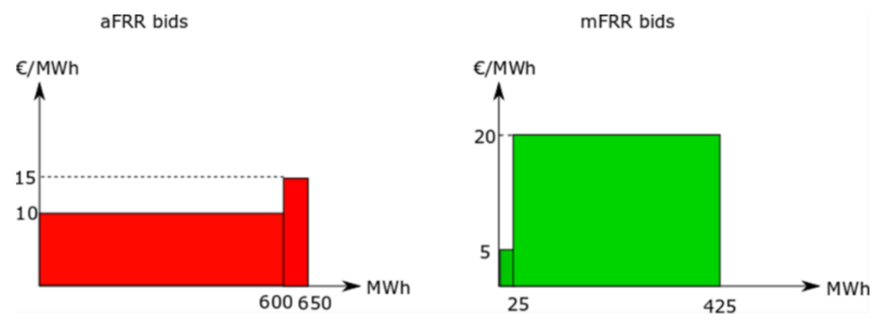


Figure 29: aFRR and mFRR bids used in Example A.1 in order to illustrate price reversal.

Consider a scenario of sequential clearing, where the aFRR market is cleared first, followed by the clearing of the mFRR market. The aFRR price in this sequential clearing context would be set by the intersection of the inelastic demand with the aFRR supply function, as indicated in the left panel of Figure 30. The resulting aFRR price is 10 €/MWh. Note that 200 MWh of the cheapest aFRR bids, as well as the second aFRR bid, both of which are not accepted in the aFRR auction, are automatically cascaded to the mFRR auction. The synthetic mFRR curve which emerges is depicted in the right panel of *Figure 30*. Intersecting this supply function with an inelastic demand of 350 MWh of mFRR, we have a clearing price of 20 €/MWh.

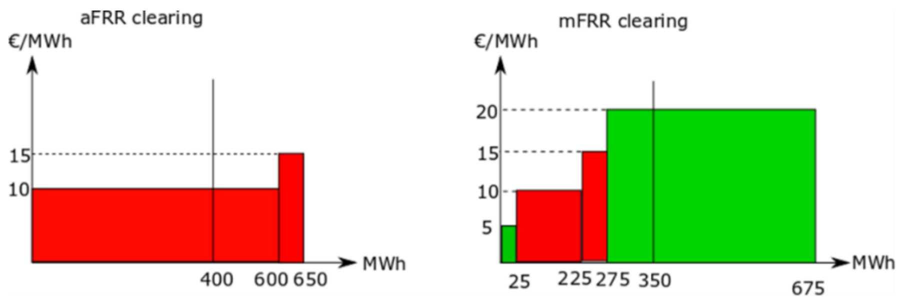


Figure 30: Clearing in a sequential context, where non-accepted aFRR bids are cascaded to the mFRR auction.

Notice the perverse incentives that are produced by this sequential design: the aFRR clearing price is lower than the mFRR clearing price! This perverse phenomenon is called **price reversal**. Price reversal is the symptom of a *non-viable market design*, because it produces a perverse incentive whereby a lower-quality product (mFRR) has a higher price than a higher-quality product (aFRR). What one would expect, and what actually happened in the California market, is that resources which are eligible to offer aFRR pretend that their technology is not qualified for aFRR. This starves the aFRR market from resources and is thus obviously inefficient since it results in a shortage of BSP supply for aFRR.

The California design was adapted in an attempt to eliminate this perverse incentive by having the aFRR price being set by the most expensive aFRR bid, whether that was cleared in the aFRR or the mFRR auction. This still does not eliminate price reversal entirely (and is anyway a patch that is not based on sound economic first principles), since the aFRR price in this example now becomes 15 €/MWh, while the mFRR price remains at 20 €/MWh.

Co-optimization overcomes this drawback of sequential clearing by recognizing endogenously, in the market clearing model, the technical interdependencies between different types of reserve. Concretely, co-optimization introduces a technical constraint which indicates that the provision of aFRR and mFRR from a single BSP are mutually exclusive, and another technical constraint which indicates that BSP resources which are technically qualified to provide aFRR are also qualified to provide mFRR. The resulting market clearing model is coherent, and the economic first principles of multi-product auctions guarantee that the resulting prices are aligned with the incentives of agents. Concretely, the application of the co-optimization model to the example of Figure 29 results in a market clearing price of 20 €/MWh for aFRR and a market clearing price of 20 €/MWh for mFRR. These prices are coherent with the incentives of agents, in the sense that the BSPs are guaranteed maximum satisfaction given the price that they are paid for the reserves that they are asked to supply by the market clearing model. It is worth noting that the KKT/optimality conditions of the market clearing model guarantee, by design, that the price of aFRR is greater than or equal to the price of mFRR. <u>Thus, by design, the co-optimization model guarantees that price reversal cannot occur</u>.

■

Appendix B: Validation of O3 Against Stylized Test Sets

In Table 15 we present the aggregate results of all proposed methods for handling the deterministic requirement against a battery of test cases. Note that column (O1) for some of these instances was already reported in [5]. We notice that our claimed results about the relative conservativeness of the different models and their hub dependency are confirmed.

	SP	(O1)	(O2)	(O3)	(No-DR)
Ex1.dat	9,765	9,765	9,765	9,765	9,765
Ex2.dat	7,814	7,814	7,814	7,814	7,814
Ex3.dat	7,812.1	7,812.1	7,812.1	7,812.1	7,812.1
Ex4.dat	147,600	147,600	147,600	147,600	147,600
Ex4.dat with L as hub	147,600	147,600	147,600	147,600	147,600
ExB1.dat	249,500	158,625	249,500	249,500	249,500
ExB2.dat (B1 with B as hub)	249,500	249,500	249,500	249,500	249,500
ExB3.dat (B1 with A as hub)	249,500	128,250	249,500	249,500	249,500
ExE1.dat (par. 5.1 [3])	167,500	157,500	167,500	167,500	167,500
ExE2.dat (E1 with B as hub)	167,500	167,500	167,500	167,500	167,500
ExE3.dat (E1 with A as hub)	167,500	127,500	127,500	167,500	167,500
Par. 5.3 [3]	170,000	170,000	170,000	170,000	170,000
Par. 5.3 [3] with B as hub	170,000	170,000	170,000	170,000	170,000
Par. 5.3 [3] with A as hub	170,000	135,000	135,000	170,000	170,000
Par. 5.6 [3]	144	144	144	144	160
Par. 5.6 [3] with B as hub	144	144	144	144	160
Par. 5.6 [3] with A as hub	144	72	72	144	160
Asymmetric	176,000	168,000	168,000	176,000	176,000
Asymmetric with B as hub	176,000	176,000	176,000	176,000	176,000
Asymmetric with A as hub	176,000	176,000	176,000	176,000	176,000

Table 15: Welfare (in €) of different methods on different examples.

Note that the example of paragraph 5.6 of [3] indicates that the feasible set of (O2) may not be a subset of SP.

Additional checks

In order to further check the validity of our conclusions, we consider two extensions:

- A version of the example of paragraph 5.3 of [3] with asymmetric PTDFs, whereby the reactance of lines AB, BC and AC obeys the ratio 3:2:1. This is indicated in Table 15 as "Asymmetric".
- A barrage of randomized examples. We proceed to explain the randomization procedure below.

The idea of the randomization procedure is to generate instances with plausible data in the range of values that are used in example 5.3 of [3]. In particular, we will consider a triangular network that is constructed as follows:

- ATC capacities are drawn uniformly from 0 to 2000 MWh for each line.
- Susceptances are sampled randomly from 0 to 1 for each line.
- Each zone has an energy seller and an energy buyer.
- Each zone has a BSP BC supply offer and a TSO BC demand offer.
- Each BSP and each energy supplier in a given zone are multilaterally linked.
- Each energy buyer is sampled uniformly and independently from 0 to 3200 MWh.
- Each TSO BC demand is sampled uniformly and independently from 0 to 4000 MWh.
- Each energy offer is uniformly and independently distributed between 0 and -6000 MWh.
- Each BSP BC supply offer is the maximum of the corresponding energy offer and a random variable that is distributed uniformly and independently between 0 and -6000 MWh.
- Each energy buyer bids uniformly and independently between 0 and 200 €/MWh.
- Each TSO demand bids uniformly and independently between 0 and 160 €/MWh.
- Each energy supplier bids uniformly and independently between 0 and 20 €/MWh.
- Each BSP supplier bids uniformly and independently between 0 and 5 €/MWh.

We run the code against 100 randomly generated samples. We find that the relative conservativeness of the models behaves as expected, and so do hub dependencies. It is also interesting to note that there are instances where the O3 model is strictly less than the SP model, demonstrating that O3 is a strictly conservative approximation of the deterministic requirement (23 out of 100 instances).

Appendix C: Co-Optimization with Bid Linking & CZC allocation: Primal/Dual and KKT Conditions

Introduction

This appendix contains some theoretical observations over co-optimization based on an oversimplified specific example. The intention of using such a specific simplified example is purely for pedagogical purposes, and all the conclusions specified below can be generalized to more advanced setups.

The content provided here is purely theoretical and is completely unrelated to any algorithm or other solving methods, hence is not related to Euphemia or other algorithms. Rather, the intention is to disentangle the links between some essential mathematical optimization concepts and their economic interpretability, especially for what concerns the BC deterministic requirement (i.e. how are congestions and congestion price differentials related in presence of cross zonal BC allocation for a simple ATC interconnector) and the bid linking requirement (i.e. how does the bid linking constraint impact bid's profitability).

The key objective of this appendix is to show the intrinsic relation between volumes (i.e. bid acceptance and flows) and prices (market clearing prices, surpluses, and congestion revenues) that occur in welfare maximization problems. Therefore, the paper firstly describes the primal "volume problem" welfare maximization problem, its "price problem" counterpart, and the (KKT) relations that link volumes and prices. It then notably tries to highlight the fact that prices cannot be determined independently of the bids' acceptance.

Problem description

In this appendix, we have considered a simplified situation where two bidding areas (FR and NL) are coupled through a unique ATC line of capacity ATC_{NLtoFR} and ATC_{FRtoNL}.

In FR, there is a demand for energy (day-ahead market) of Q_{FR}^{l} at a price of P_{FR}^{l} and for balancing capacity of QR_{FR}^{l} at a price of PR_{FR}^{l}. FR also sees a supply of Q_{FR}^{s} of energy at a limit price of P_{FR}^{s} and a supply of balancing capacity of QR_{FR}^{s} at a limit price of PR_{FR}^{s}.

In NL, one generation asset is willing to supply either Q_{NL}^{s} of energy at a price of P_{NL}^{s} or QR_{NL}^{s} of balancing capacity at a price of PR_{NL}^{s}. The asset in NL thus needs to use linked bids.

By convention, supply quantities have negative values ($Q^{s} < 0$) and demand quantities have positive values ($Q^{l} > 0$).

The notation convention is illustrated in Figure 31 below.

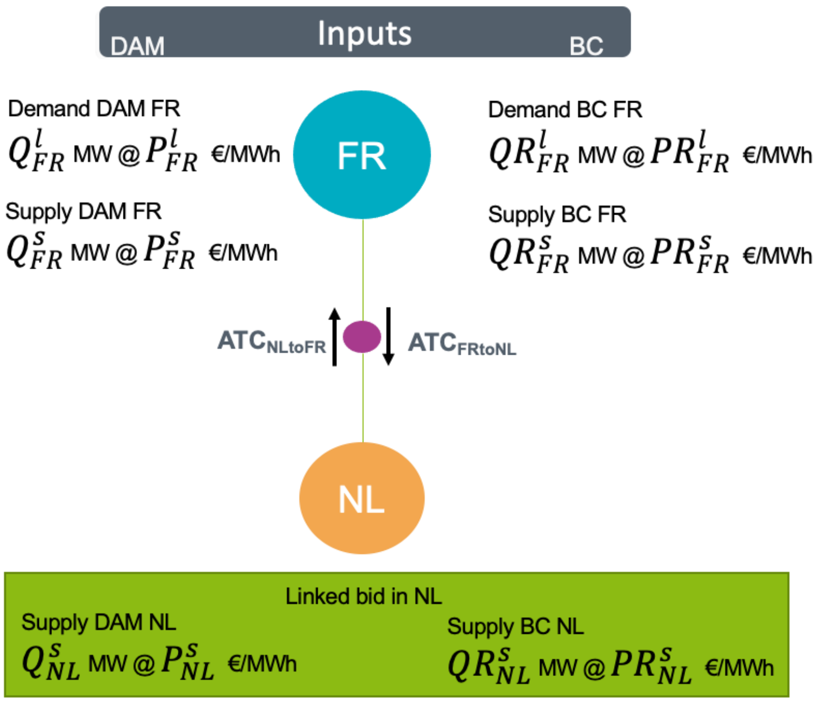


Figure 31: Notation convention in Appendix C.

One-Step multilateral bid linking model

In this section, we will consider the use of a one-step co-optimization program with multilateral bid linking between the two NL supply offers for BC and DA energy. This is a welfare co-optimization model, i.e. it optimizes the sum of the welfare of the DA market and the welfare of the BC market.

The primal and dual optimization programs along with the KKT conditions associated with this problem are presented in the following sections below.

Primal problem

Model

The primal optimization model is represented by the following equations:

$$(A1_P): max_{x,xR,f,fR}\ P^l_{FR} \cdot Q^l_{FR} \cdot x^l_{FR} + PR^l_{FR} \cdot QR^l_{FR} \cdot xR^l_{FR} + P^s_{FR} \cdot Q^s_{FR} \cdot x^s_{FR} + PR^s_{FR} \cdot QR^s_{FR} \cdot xR^s_{FR} + P^s_{NL} \cdot Q^s_{NL} \cdot x^s_{NL} + PR^s_{NL} \cdot QR^s_{NL} \cdot xR^s_{NL} \quad (1)$$

$$-f_{NLtoFR} + Q^l_{FR} \cdot x^l_{FR} + Q^s_{FR} \cdot x^s_{FR} = 0 \qquad (\varepsilon_{FR}) \qquad (2)$$

$$fR_{FRtoNL} - fR_{NLtoFR} + QR^l_{FR} \cdot xR^l_{FR} + QR^s_{FR} \cdot xR^s_{FR} = 0 \qquad (\varepsilon R_{FR}) \qquad (3)$$

$$f_{NLtoFR} + Q^s_{NL} \cdot x^s_{NL} = 0 \qquad (\varepsilon_{NL}) \qquad (4)$$

$$fR_{NLtoFR} - fR_{FRtoNL} + QR^s_{NL} \cdot xR^s_{NL} = 0 \qquad (\varepsilon R_{NL}) \qquad (5)$$

$$f_{NLtoFR} + fR_{NLtoFR} \leq ATC_{NLtoFR} \qquad (\alpha_{NLtoFR}) \qquad (6a)$$

$$-f_{NLtoFR} + fR_{FRtoNL} \leq ATC_{FRtoNL} \qquad (\alpha_{FRtoNL}) \qquad (6b)$$

$$fR \geq 0 \qquad (6c)$$

$$x^S_{NL} + xR^s_{NL} \leq 1 \qquad (\lambda) \qquad (7)$$

$$0 \leq x, xR \leq 1 \qquad (\beta, \beta R) \qquad (8)$$

Interpretation

Let us describe these equations in plain English:

- Equation (1) is the standard welfare maximization objective, where we want to determine the optimal acceptance ratios of each bid, denoted with x and xR variables. It sums the economic contributions of all bids (where demand bids have positive contributions and supply bids negative contributions since $Q^l > 0$ and $Q^s < 0$)

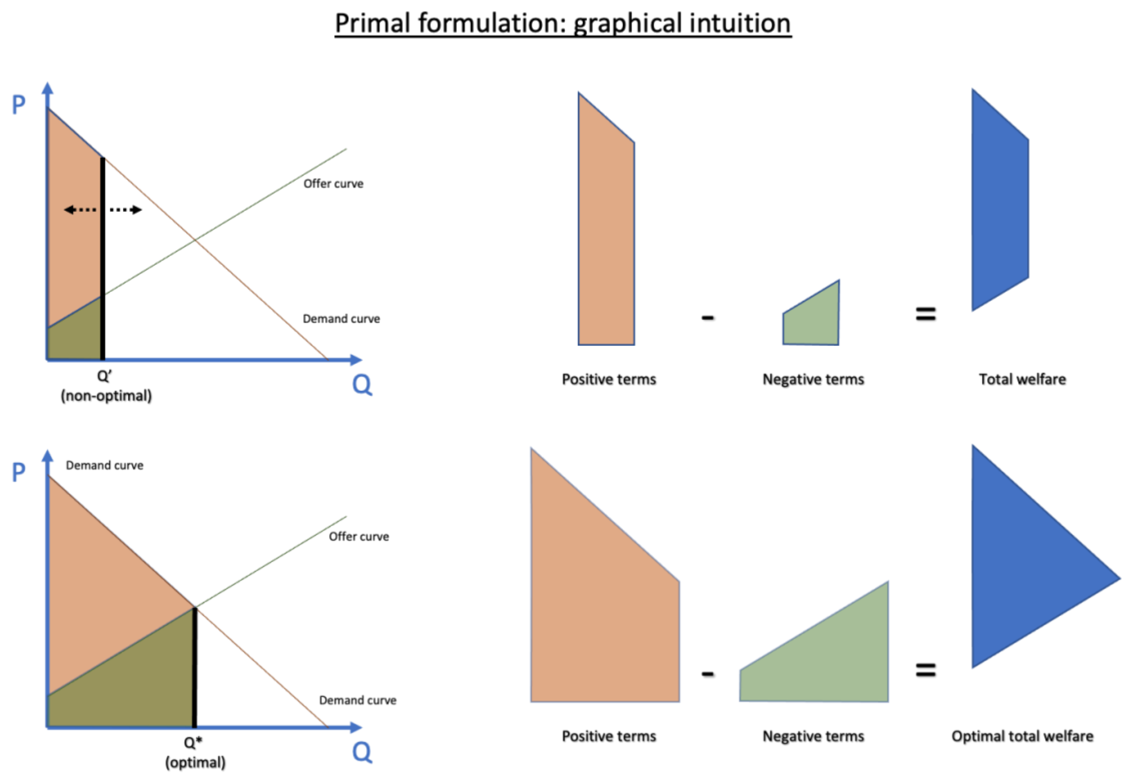


Figure 32: A graphical intuition of the primal formulation is the maximization of the integral under the demand curve (orange) minus the integral under the supply curve (green) with the constraint that total demand equals total supply. We thus move a vertical line over the x-axis, and find out that the largest difference between these two surfaces (blue) is to be found at the curves' intersection.

- Equations (2) and (4) are the energy equilibrium constraints, while Equations (3) and (5) are the BC equilibrium constraints. They ensure that accepted supply minus accepted demand equal to the interconnector's allocation. Though, there exists an important difference between interconnector allocation for energy and for BC. For energy, there can only be a single "flow setpoint" over the interconnector, represented by a single variable f_{NLtoFR} which can take positive or negative values. For BC, the deterministic requirement (cfr. Infra) imposes to use two distinct variables fR_{FRtoNL} and fR_{NLtoFR} that represent the possible "flow ranges" that must be reserved for any possible volume of BC energy activation. Such unidirectional "flow ranges" must have non-negative values (Equation (6c)) which cannot net out each other.

- Equations (6a) and (6b) limit the sum of the energy "flow setpoint" and BC "flow ranges" to the ATC limit. These equations mimic the fact that – contrary to the energy "flow setpoint"- the BC "flow ranges" cannot be netted because they are uncertain. It is the reason why the BC flow range in the opposite direction of the ATC is not included in the formula.

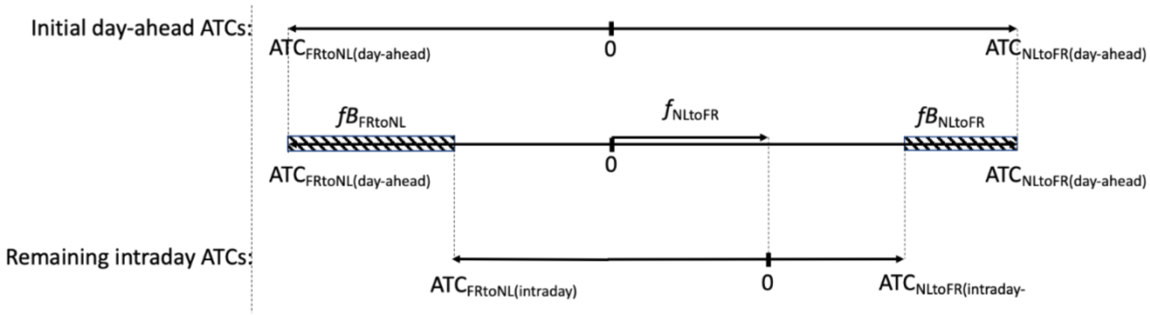


Figure 33: The graphical intuition of the difference between cross-zonal allocation for BC and for energy, is that a BC flow range (i.e. fBFRtoNL and fBNLtoFR) occupy a "space" of the CZC domain – which thus must be non-negative in each direction and cannot be netted; unlike the energy flow setpoint (i.e. f_{NLtoFR}) which is appropriately represented with a single variable that can be negative as a flow in one direction necessarily frees up CZC in the opposite direction. The figure depicts how the residual intraday ATCs are deducted from the allocations of BC and energy flows.

- Equation (7) represents the bid linking of the orders in NL, which takes the form of an exclusivity constraint: the sum of the acceptance ratios of both products cannot exceed 100%. This is a "multilateral bid linking" constraint, in the sense that no product has any type of priority over the other one.

- Equation (8) states that the acceptance ratios cannot individually exceed 100%.

In optimization jargon, this main optimization program is called a "primal problem" (in this case a Linear Program, or LP). Solving this problem (with whatever mathematical technique) provides by construction the welfare optimal feasible solution: it seeks among all solutions defined as feasible the one that has the largest objective function.

Noticeably, this primal problem only contains variables which are related to quantities (it contains only either acceptance ratios, which – once multiplied by their related Q values – give the accepted volumes for each bid, or flow variables). Therefore, we often refer to the welfare maximization problem as a "volume problem". Next to each constraint, we indicate on the right side its "dual variable", which we introduce below.

Dual problem

The dual problem is an alternative formulation of the same problem as the primal one. For convex problems (which is the case here as we describe a linear primal optimization problem), the optimal objective function value of the primal and the dual formulations are equal. The dual problem uses the same input parameters as the primal problem but in a specific alternative way (i.e. determined with some matrix inversion), and uses different variables (i.e. the dual variables, also sometimes called "shadow prices" or "Lagrange multipliers").

We will now explain the importance of these "math magics", justified by unquestionable "duality theory", for what concerns economic interpretation of the primal problem and underlying pricing equilibrium aspects. Such "math magics" are in fact optimization theorems which are not only factually unquestionable – they also lead to the appreciable economic properties. This is why N-SIDE has continuously challenged the fact to dissociate bids acceptance (i.e. primal optimum solutions) from the resulting prices (i.e. dual optimal solutions).

Note however that this theory no longer fully holds in case binary variables are used (e.g. for indivisible bids), as such variables imply non-convexities. Indivisibilities and other non-convex requirements are neglected in the remainder of this appendix – even though they will have to be fully addressed in the real-life co-optimization problem. In Euphemia, all the properties applicable to the linear relaxation of the problem (i.e. a convex approximation of the real-problem problem) have been applied with one single exception: indivisible bids may be paradoxically rejected. Our expectation has been that a similar approach (i.e. basing the market rules and properties of the results on the same theory applicable to convex welfare maximization problems and allowing the same exception to cope with non-convexities) will be the basis of the co-optimization market rules.

Below is the dual formulation of the primal problem stated above. We indicate also on the right of each constraint its related "primal variable".

Model

$$(A1_D): min_{\varepsilon,\varepsilon R,\alpha,\lambda,\beta,\beta R} \; ATC_{NLtoFR} \cdot \alpha_{NLtoFR} + ATC_{FRtoNL} \cdot \alpha_{FRtoNL} + \lambda + \beta^{l}_{FR} + \beta R^{l}_{FR} + \beta^{s}_{FR} + \beta R^{s}_{FR} + \beta^{s}_{NL} + \beta R^{s}_{NL} \tag{21}$$

$$-\varepsilon_{FR} + \varepsilon_{NL} + \alpha_{NLtoFR} - \alpha_{FRtoNL} = 0 \qquad (f_{NLtoFR}) \tag{22}$$

$$-\varepsilon R_{FR} + \varepsilon R_{NL} + \alpha_{NLtoFR} \geq 0 \qquad (fR_{NLtoFR}) \tag{23a}$$

$$\varepsilon R_{FR} - \varepsilon R_{NL} + \alpha_{FRtoNL} \geq 0 \qquad (fR_{FRtoNL}) \tag{23b}$$

$$QR^{s}_{NL} \cdot \varepsilon R_{NL} - PR^{s}_{NL} \cdot QR^{s}_{NL} + \beta R^{s}_{NL} + \lambda \geq 0 \qquad (xR^{s}_{NL}) \tag{24}$$

$$Q^{s}_{NL} \cdot \varepsilon_{NL} - P^{s}_{NL} \cdot Q^{s}_{NL} + \beta^{s}_{NL} + \lambda \geq 0 \qquad (x^{s}_{NL}) \tag{25}$$

$$QR^{s}_{FR} \cdot \varepsilon R_{FR} - PR^{s}_{FR} \cdot QR^{s}_{FR} + \beta R^{s}_{FR} \geq 0 \qquad (xR^{s}_{FR}) \tag{26}$$

$$Q^{s}_{FR} \cdot \varepsilon_{FR} - P^{s}_{FR} \cdot Q^{s}_{FR} + \beta^{s}_{FR} \geq 0 \qquad (x^{s}_{FR}) \tag{27}$$

$$QR^{l}_{FR} \cdot \varepsilon R_{FR} - PR^{l}_{FR} \cdot QR^{l}_{FR} + \beta R^{l}_{FR} \geq 0 \qquad (xR^{l}_{FR}) \tag{28}$$

$$Q_{FR}^{l} \cdot \boldsymbol{\varepsilon}_{FR} - P_{FR}^{l} \cdot Q_{FR}^{l} + \boldsymbol{\beta}_{FR}^{l} \geq 0 \qquad (x_{FR}^{l}) \qquad (29)$$

$$\boldsymbol{\beta}, \boldsymbol{\beta R}, \boldsymbol{\alpha}, \boldsymbol{\lambda} \geq 0 \qquad (30)$$

Interpretation

- Equation (21) is the dual objective function, which contains variables α, β & λ. We will show below that $\alpha \cdot ATC$ represents the congestion revenue and that β represents the surplus (i.e. the profit) made by accepted orders. We will then specifically focus on the interpretation of λ, which is the dual variable of the bid linking constraint and embeds the "opportunity cost of (multilateral) bid linking". The constraints also include variables ε and εR that respectively represent the market clearing price of energy and balancing capacity.

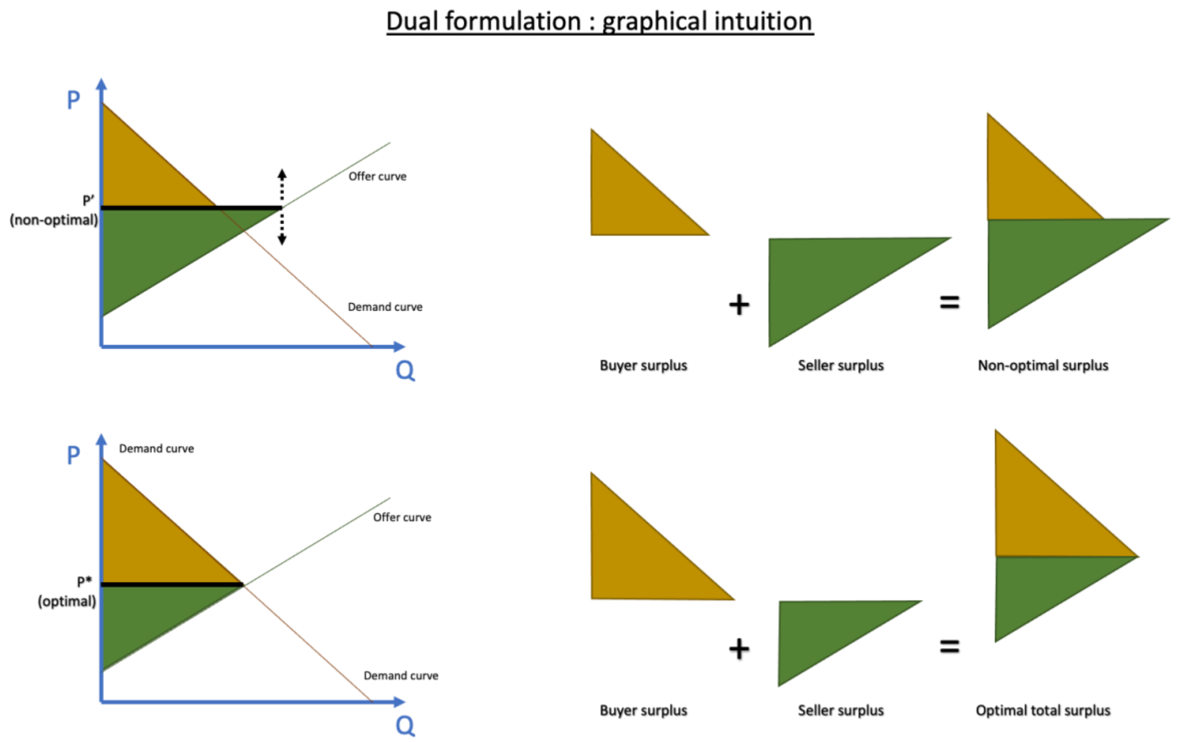


Figure 34: Illustration of a graphical interpretation of this dual formulation. The dual optimization program is equivalent to moving a horizontal line along the y-axis, which represents the market clearing prices. For a given clearing price level, we sum the profits of all "in the money bids", which is represented by two positive surfaces (1) between our horizontal line and the offer curve below the axis – referred to as "supply surplus" (green) and (2) between our horizontal line and the demand curve above the axis – referred to as "demand surplus" (yellow). We find that the smallest surface is obtained at the curves' intersection: at optimality, where the total welfare equals the sum of these surpluses. The congestion revenue (which equal $\alpha \cdot ATC$ in the objective function) is not represented in this figure but would also qualify as the "grid profit/surplus".

- Equation (30) imposes that the variables which are minimized in the objective function are non-negative, which implies that neither bids nor the interconnector will ever make losses.

- Equation (22) defines the cross-zonal energy price differential as the difference between the dual variables of the ATC constraints $(\varepsilon_{NL} - \varepsilon_{FR}) = (\alpha_{FRtoNL} - \alpha_{NLtoFR})$. This is typical and known from the current energy-only day-ahead market. One important difference however compared to today's situation is that – due to BC allocation over the interconnector – ATC constraints in opposite directions can be concurrently binding, i.e. it is possible that $\alpha_{FRtoNL} > 0$ and $\alpha_{NLtoFR} > 0$ at the same time.

- Equations (23a) & (23b) give a lower bound to the CZC value based on the cross-zonal BC price differentials: the CZC value in a given direction is at least equal to the positive BC price spread in this direction, i.e. $(\varepsilon R_{FR} - \varepsilon R_{NL}) \leq \alpha_{NLtoFR}$ and $(\varepsilon R_{NL} - \varepsilon R_{FR}) \leq \alpha_{FRtoNL}$

- Equations (24) to (29) provide a lower bound to the surplus variables of price compatible bids. For example, from Equation (29), we know that if $\varepsilon_{FR} > P_{FR}^{l}$ (i.e. the demand order is "in the money", meaning that its price is compatible with the market clearing price), then its profit is positive and equals the volume of the bid multiplied by its "in the moneyness" (i.e. the difference between its limit price and the market clearing price). Otherwise (i.e. if the order is out of the money), this "floor" has a negative value and is superseded by Equation (30): bid's profit is null.

- Note that Equations (24) and (25) – which apply to the linked bid – share an additional common term λ in their profit functions. This is an important implication of bid linking which we discuss below.

KKT Conditions

Model

The KKT conditions representing the problem are given by the following set of equations:

$$\boldsymbol{\alpha_{NLtoFR} \geq 0 \perp f_{NLtoFR} + fR_{NLtoFR} \leq ATC_{NLtoFR}} \tag{10a}$$

$$\boldsymbol{\alpha_{FRtoNL} \geq 0 \perp -f_{NLtoFR} + fR_{FRtoNL} \leq ATC_{FRtoNL}} \tag{10b}$$

$$\boldsymbol{fR_{NLtoFR} \geq 0 \perp -\varepsilon R_{FR} + \varepsilon R_{NL} + \alpha_{NLtoFR} \geq 0} \tag{10c}$$

$$\boldsymbol{fR_{FRtoNL} \geq 0 \perp \varepsilon R_{FR} - \varepsilon R_{NL} + \alpha_{FRtoNL} \geq 0} \tag{10d}$$

$$\boldsymbol{\lambda \geq 0 \perp x_{NL}^{s} + xR_{NL}^{s} \leq 1} \tag{11}$$

$$\boldsymbol{\beta \geq 0 \perp x \leq 1} \tag{12}$$

$$\boldsymbol{x_{FR}^{l} \geq 0 \perp Q_{FR}^{l} \cdot \varepsilon_{FR} - P_{FR}^{l} \cdot Q_{FR}^{l} + \beta_{FR}^{l} \geq 0} \tag{13}$$

$$\boldsymbol{xR_{FR}^{l} \geq 0 \perp QR_{FR}^{l} \cdot \varepsilon R_{FR} - PR_{FR}^{l} \cdot QR_{FR}^{l} + \beta R_{FR}^{l} \geq 0} \tag{14}$$

$$\boldsymbol{x_{FR}^{s} \geq 0 \perp Q_{FR}^{s} \cdot \varepsilon_{FR} - P_{FR}^{s} \cdot Q_{FR}^{s} + \beta_{FR}^{s} \geq 0} \tag{15}$$

$$\boldsymbol{xR_{FR}^{s} \geq 0 \perp QR_{FR}^{s} \cdot \varepsilon R_{FR} - PR_{FR}^{s} \cdot QR_{FR}^{s} + \beta R_{FR}^{s} \geq 0} \tag{16}$$

$$x_{NL}^{s} \geq 0 \perp Q_{NL}^{s} \cdot \varepsilon_{NL} - P_{NL}^{s} \cdot Q_{NL}^{s} + \beta_{NL}^{s} + \lambda \geq 0 \tag{17}$$

$$xR_{NL}^{s} \geq 0 \perp QR_{NL}^{s} \cdot \varepsilon R_{NL} - PR_{NL}^{s} \cdot QR_{NL}^{s} + \beta R_{NL}^{s} + \lambda \geq 0 \tag{18}$$

$$-\varepsilon_{FR} + \varepsilon_{NL} + \alpha_{NLtoFR} - \alpha_{FRtoNL} = 0 \tag{19}$$

$$-f_{NLtoFR} + Q_{FR}^{l} \cdot x_{FR}^{l} + Q_{FR}^{s} \cdot x_{FR}^{s} = 0 \tag{20}$$

$$fR_{FRtoNL} - fR_{NLtoFR} + QR_{FR}^{l} \cdot xR_{FR}^{l} + QR_{FR}^{s} \cdot xR_{FR}^{s} = 0 \tag{21}$$

$$f_{NLtoFR} + Q_{NL}^{s} \cdot x_{NL}^{s} = 0 \tag{22}$$

$$fR_{NLtoFR} - fR_{FRtoNL} + QR_{NL}^{s} \cdot xR_{NL}^{s} = 0 \tag{23}$$

KKT conditions (i.e. Karush-Kuhn-Tucker conditions, also sometimes called complementary slackness conditions) are also consequent to convex optimization problems. They link the primal constraints (left of the ⊥ sign) and dual constraints (right of the ⊥ sign) values at optimality: finding a feasible solution that respects this set of equations is a third method that can be used to find the optimal solution (although there is no objective function).

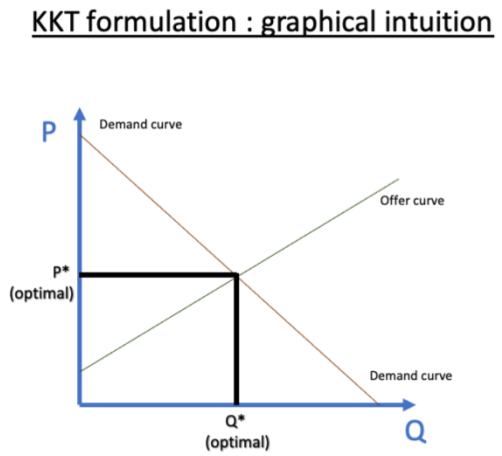


Figure 35: Graphical illustration for solving the KKT equations. Solving the KKT equations is equivalent to identifying the intersection of the offer and demand curves (i.e. the point that respects all constraints), without calculating/optimizing any surface (i.e. without the need of any objective function).

A key principle of KKT conditions is that a dual variable of an inequality constraint can only have strictly positive values in case the primal constraint is binding, as a dual variable represents the "value" of a primal constraint (i.e. how much a constraint impacts the objective function is necessarily zero for a non-binding constraint). Therefore, a dual variable is often referred to as a "shadow price" in economic problems.

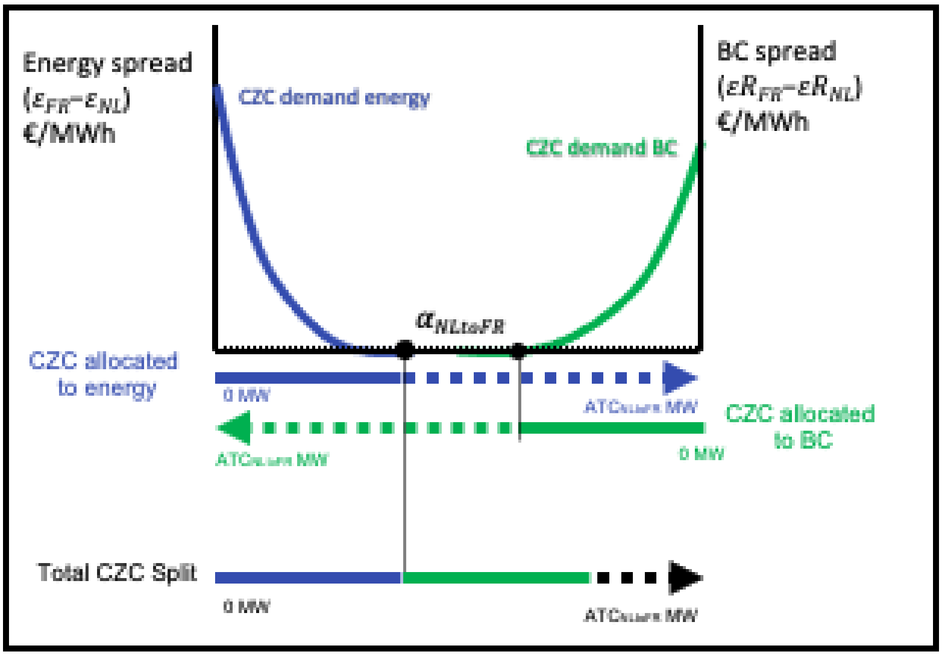
More generally, KKT conditions imply that, for specific pairs of primal and dual constraints separated by the ⊥ sign, at least one of the constraints must be binding (i.e. at least one of the two inequalities can be replaced by an equality in any optimal feasible solution).

Interpretation

We now describe how these KKT equations can be interpreted.

Congestion management

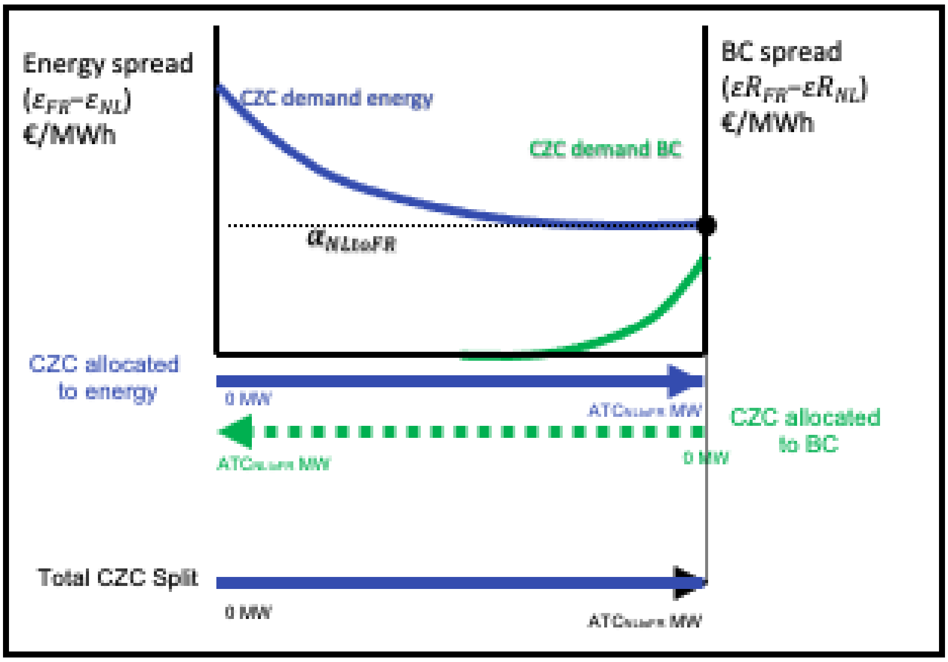
- Equation (10a) & (10b) indicate that the dual variable α (i.e. the "value" of an ATC constraint) can only have non-zero value in case the ATC constraint is binding (since the $\perp$ sign means that at least one of the two equations must be replaceable by an equality constraint). Note that, from the dual objective function, a non-zero α_{NLtoFR} or α_{FRtoNL} lead to a positive congestion revenue (as α is in this case strictly positive, and if we assume that ATCs are also positive). This is a well-known property of implicit CZC auctions: grid's profit cannot be negative.
- Equations (10c) and (10d) link the existence of non-zero BC flow ranges (fR_{NLtoFR} or fR_{FRtoNL}) to the cross-zonal BC price differences $\varepsilon R_{NL} - \varepsilon R_{FR}$, which in turn is bounded by the directional shadow price of the ATC constraints α_{NLtoFR} and α_{FRtoNL}. We now know that in case CZC is allocated to BC, CZC value equals the BC price spread.
- To get a better intuition of these constraints interaction, let us discuss various congestion patterns, and describe what can be deducted from Equations (10a), (10b), (10c), (10d) and (19):
    - To start, we can see from these equations that there will be no cross-zonal price differentials on either product in case there is no congestion in either direction: $\varepsilon_{FR} = \varepsilon_{NL}$ and $\varepsilon R_{FR} = \varepsilon R_{NL}$ if Equations (6a) and (6b) are not binding (hence if α_{NLtoFR} =$\alpha_{FRtoNL} = 0$).



If there is no congestion, there is no price difference

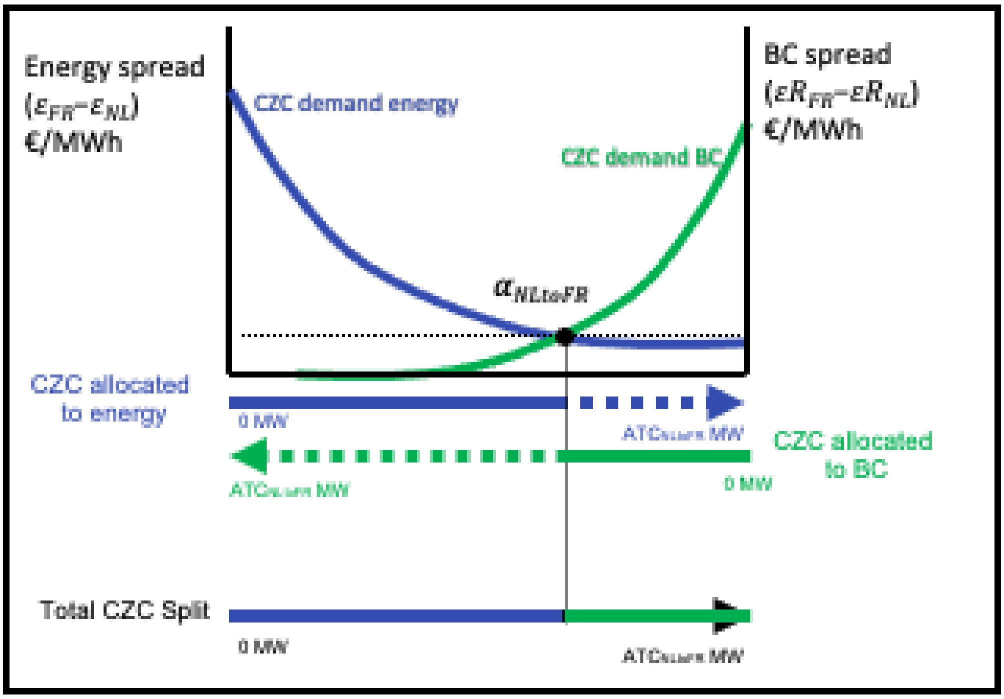
    - Let us now assume that the interconnector is congested while being fully allocated to energy, such that $f_{NLtoFR} = ATC_{NLtoFR}$ with $fR_{NLtoFR} = 0$

and $fR_{FRtoNL} = 0$. As there is only congestion in one direction, we know that $\varepsilon_{FR} - \varepsilon_{NL} = \alpha_{NLtoFR}$ which is a well-known energy-only implicit auction property implied by Equation (19). We can now deduct that the cross-zonal BC price differential is of the same sign and smaller than the energy price differential, i.e. $0 \leq \varepsilon R_{FR} - \varepsilon R_{NL} \leq \varepsilon_{FR} - \varepsilon_{NL} = \alpha_{NLtoFR}$. This is economically efficient: CZC has been entirely allocated to energy, i.e. the most profitable product in terms of CZC allocation (measured by its cross-zonal price spread). The BC cross-zonal price differential can however not be of the opposite sign, as CZC would then have been concurrently allocated to BC in the opposite direction.



If CZC is fully allocated only to energy, then the energy cross-zonal price spread is at least equal to the positive BC cross-zonal price spread

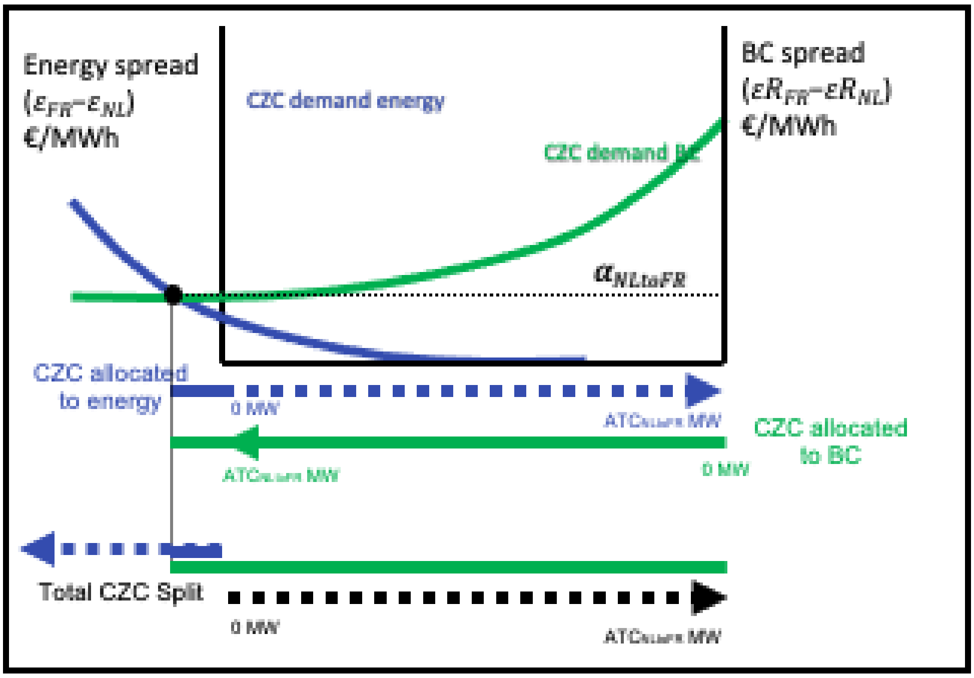
- If the interconnector is allocated to both energy and BC in the same direction ($f_{NLtoFR} + fR_{NLtoFR} = ATC_{NLtoFR}$ with $f_{NLtoFR} > 0$, $fR_{NLtoFR} > 0$), the CZC is split such that $\varepsilon R_{FR} - \varepsilon R_{NL} = \varepsilon_{FR} - \varepsilon_{NL} = \alpha_{NLtoFR}$. Again, this makes sense economically: if one price differential would be larger than the other one, then the CZC split should allocate more to this product; a mixed BC and energy allocation in the same direction thus must mean that both products have reached equal cross-zonal value.



If CZC is allocated to both BC and energy, then the energy cross-zonal price spread is equal to the BC cross-zonal price spread

- If the interconnector is fully allocated to BC in one direction[21] $(fR_{NLtoFR} + f_{NLtoFR} = ATC_{NLtoFR}, with\ f_{NLtoFR} \leq 0)$, then, since $\varepsilon_{FR} - \varepsilon_{NL} = \alpha_{NLtoFR} - \alpha_{FRtoNL}$, either the cross-zonal energy price differential is equal to the BC one ($\varepsilon R_{FR} - \varepsilon R_{NL} = \varepsilon_{FR} - \varepsilon_{NL} = \alpha_{NLtoFR}$ if $\alpha_{FRtoNL} = 0$), or there is a congestion due to energy allocation in the opposite direction (if $\alpha_{FRtoNL} \geq 0$ then $-f_{NLtoFR} = ATC_{FRtoNL}$). This is again determined by economic efficiency, but is a significant difference compared to today's situation: **while nowadays energy flows over ATC-based interconnectors always go from low price to higher (or equal) price zones, this may not always be the case under co-optimization**. Indeed, as long as the energy cross-zonal spread is smaller than the BC cross-zonal spread, it remains optimal to release CZC with energy to enable further allocation to BC – including if this implies flowing in opposite direction of the energy price spread. Note that by construction, the congestion revenues nonetheless always remain positive – despite apparent "adverse flows".

21 Note that we include in this BC congestion pattern the energy flow in the opposite direction so as to account for netting



If CZC is allocated only BC, then the energy cross-zonal price spread
is either equal to the BC cross-zonal price spread (no congestion in opposite direction)
Or lower than the BC cross-zonal price spread (congestion in the opposite direction)

- Let us now redo a similar exercise, but – instead of observing congestion patterns to deduct price differentials – we observe price differentials to deduct the congestion patterns:
  - $\varepsilon_{FR} - \varepsilon_{NL} = \varepsilon R_{FR} - \varepsilon R_{NL} = 0$

    If the cross-zonal price spread is null for the two products, the only thing which can be deducted is that the ATC constraints are not binding. It is not possible to deduct the flow directions in this case.

  - $\varepsilon_{FR} - \varepsilon_{NL} > \varepsilon R_{FR} - \varepsilon R_{NL} > 0$

    If the energy price spread is larger than the BC price spread – both being of the same sign, we can be sure that the CZC split has allocated capacity only to energy in one direction ($f_{NLtoFR} = ATC_{NLtoFR}$ and $fR_{NLtoFR} = 0$).

  - $\varepsilon R_{FR} - \varepsilon R_{NL} > \varepsilon_{FR} - \varepsilon_{NL} > 0$
    If the BC price spread is larger than the energy price – both being of the same sign – we can be sure that the energy flow is allocated "adversely" (i.e. against the energy price spread) up to congestion (which implies that $fR_{NLtoFR} = ATC_{NLtoFR} + ATC_{FRtoNL}$ and $f_{NLtoFR} = -ATC_{FRtoNL}$). This is because such an energy adverse flow – although somehow made "at a loss" – releases further CZC that can be allocated « in the right direction » to BC such that the loss in energy is more than compensated by the gain on BC. **This is a property worth noting as it is not necessarily intuitive for those solely familiar with energy CZC allocation.**

  - $\varepsilon_{FR} - \varepsilon_{NL} > 0$ and $\varepsilon R_{FR} - \varepsilon R_{NL} < 0$:
    If the energy price spread and the BC price spread are of opposite signs, then the CZC is fully allocated to energy in one direction and to BC in the opposite

direction ($f_{NLtoFR} = ATC_{NLtoFR}$ and $fR_{FRtoNL} = ATC_{FRtoNL}$).

- $\varepsilon_{FR} - \varepsilon_{NL} > 0$ and $\varepsilon R_{FR} - \varepsilon R_{NL} = 0$
  If there is an energy price spread, but no BC price spread, there is a unidirectional congestion "in the right direction". It is however unknown if the CZC is allocated only in one or in both directions (we know that $f_{NLtoFR} + fR_{NLtoFR} = ATC_{NLtoFR}$ and $fR_{NLtoFR} \geq 0 \perp fR_{FRtoNL} \geq 0$).
- $\varepsilon R_{FR} - \varepsilon R_{NL} > 0$
  Similarly, if there is an BC price spread, but no energy price spread, there is a unidirectional congestion "in the right direction". It is however unknown if the CZC is allocated only in one or in both directions ($f_{NLtoFR} + fR_{NLtoFR} = ATC_{NLtoFR}$ with either $f_{NLtoFR} \geq 0$ or $fR_{NLtoFR} \leq 0$)

Bid management

- Acceptance & surplus of regular orders: (Let us focus on the energy demand order in FR.)
  - R1: If this order is fully rejected ($x^l_{FR} = 0$), then β^l_{FR} must equal zero from Equation (12). Knowing this, we deduct from Equation (13) that $P^l_{FR} \leq \varepsilon_{FR}$. .. In business words: a fully rejected order cannot be in the money.
  - R2: Conversely, if the order is strictly out of the money $P^l_{FR} < \varepsilon_{FR}$, then the right/dual part of Equation (13) cannot be binding, and so the left/primal part is binding and $x^l_{FR} = 0$. In business words, an order that is strictly out of the money must be fully rejected.
  - R3: Similarly, if the same order is fully accepted $x^l_{FR} = 1$, then from Equation (13), $\beta^l_{FR} = -(\varepsilon_{FR} - P^l_{FR}) \cdot Q^l_{FR}$, and since $\beta^l_{FR} \geq 0$, we know that $P^l_{FR} \geq \varepsilon_{FR}$. In business terms, fully accepted orders cannot be out of the money.
  - R4: Conversely, if the order is strictly in the money ($P^l_{FR} > \varepsilon_{FR}$), then it is not possible to satisfy Equation (28) without a positive β^l_{FR} value, which implies (from Equation (12)) that the order must be fully accepted. Or: an order that is strictly in the money must be fully accepted. Note also that it is only in this case that β^l_{FR} can take positive values, i.e. only **strictly** in the money orders can make profits.
  - R5: If an order is partially accepted ($0 < x^l_{FR} < 1$), then β^l_{FR} must equal zero from Equation (12) and thus from Equation (13) $Q^l_{FR} \cdot \varepsilon_{FR} - P^l_{FR} \cdot Q^l_{FR} + 0 = 0$; or $P^l_{FR} = \varepsilon_{FR}$. In plain English: an order partially accepted (often referred to as the "marginal order") sets the market clearing price.
  - We just proved that the standard uniform market clearing rules, which ensure coherence of orders' acceptances and their profitability, are natural properties of welfare maximization problems. The same reasoning applies to Equations

(14), (15) & (16), for the other regular orders present in our example. More generally, such "uniform market clearing rules" can be summarized as:

- **R1. Fully rejected orders => cannot be in the money**
- **R2. Strictly out of the money orders => must be fully rejected**
- **R3. Fully accepted orders => cannot be out of the money**
- **R4. Strictly in the money orders => must be fully accepted**
- **R5. Partially accepted orders => are "at the money" & set the clearing price**

- We describe below why such rules – which strictly always apply to regular orders - don't necessarily apply in the same way to linked bids.

The tables below summarize these rules (for the case of a regular demand order in FR)

Fully rejected order $\boldsymbol{x(R)^s_{NL} = 0}$	Partially accepted order $\boldsymbol{0 < x^s_{NL} < 0}$	Fully accepted order $\boldsymbol{x^s_{NL} = 1}$
$\boldsymbol{\varepsilon(R)_{FR} \geq P(R)^l_{FR}}$ Cannot be in the money (R1)	$\boldsymbol{\varepsilon(R)_{FR} = P(R)^l_{FR}}$ sets the clearing price (R5)	$\boldsymbol{\varepsilon(R)_{FR} \leq P(R)^l_{FR}}$ Cannot be out of the money (R3)

Out of the money order $\boldsymbol{\varepsilon(R)_{FR} > P(R)^l_{FR}}$	At the money order $\boldsymbol{\varepsilon(R)_{FR} = P(R)^l_{FR}}$	In the money order $\boldsymbol{\varepsilon(R)_{FR} < P(R)^l_{FR}}$
$\boldsymbol{x(R)^l_{FR} = 0}$ Must be fully rejected (R2)	$\boldsymbol{0 \leq x(R)^l_{FR} \leq 1}$ Can be of any acceptance	$\boldsymbol{x(R)^l_{FR} = 1}$ Must be fully accepted (R4)

- Acceptance & surplus of linked bids
  - We now follow the same logic to deduct similar "standard uniform market clearing rules" that apply to multilateral bid linking. Let us start by identifying some key differences between regular orders and linked bids:
    - The bid linking constraint of Equation (7), combined with the KKT condition of Equation (12), implies that two linked bids cannot concurrently have strictly positive "surplus" values.

$$\beta^s_{NL} \geq 0 \perp \beta R^s_{NL} \geq 0 \tag{31}$$

    - Further, compared to Equations (13)-(16) that we discussed above, and which apply to regular orders, equations (17) & (18) include the shadow price λ of the bid linking constraint (7).

- Let us somewhat reformulate these two last equations to isolate $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL})$ and $QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL})$ that define the "profitability[22]" of a (linked) order if it is accepted:

$$x_{NL}^{s} \geq 0 \ \perp Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) \leq \beta_{NL}^{s} + \lambda \qquad (17')$$

$$xR_{NL}^{s} \geq 0 \ \perp QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL}) \leq \beta R_{NL}^{s} + \lambda \qquad (18')$$

- R6: If the profitability of at **most** one of two linked bids is positive (i.e. if at least one linked order is not in the money), then the standard uniform market clearing rules (listed as R1-R5 above) fully apply for the two linked bids. This is firstly because the β of at least one of the linked orders is necessarily zero, hence Equation (31) is de facto satisfied, and secondly because the bid linking constraint is actually not binding[23], hence its shadow price is null ($\lambda = 0$). So nothing is in fact different than with regular orders if at least one of the linked bid is out of the money. This is an important observation for what concerns bid linking: **the bid linking constraint is only binding and impactful in case the two linked bids are in (or at) the money**. This is economically logic: the bid linking constraint is only necessary in case it is unclear which of the 2 linked bids should be accepted.

- Otherwise, i.e. if the linked orders are both in (or at) the money, $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) \geq 0$ and $QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL}) \geq 0$, we need to distinguish three cases

  - $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) > QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL})$:
    If the energy bid is more profitable than the BC bid, resolving equations (17') and (18') while respecting Equation (31) imply that $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) = \beta_{NL}^{s} + \lambda$ and $QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL}) = \lambda$, which in turn implies that $x_{NL}^{s} = 1$ and $xR_{NL}^{s} = 0$.

  - $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) < QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL})$:

    If the energy bid is less profitable than the BC bid, resolving equations (17') and (18') while respecting Equation (31) imply that $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) = \lambda$ and $QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL}) = \beta R_{NL}^{s} + \lambda$, which in turn implies that $x_{NL}^{s} = 0$ and $xR_{NL}^{s} = 1$.

  - R7: In economic terms, we have shown that **a welfare optimal solution will always select among linked bids the one that is the most profitable, if there is any[24].**

22 Recall that supply bids have negative quantities

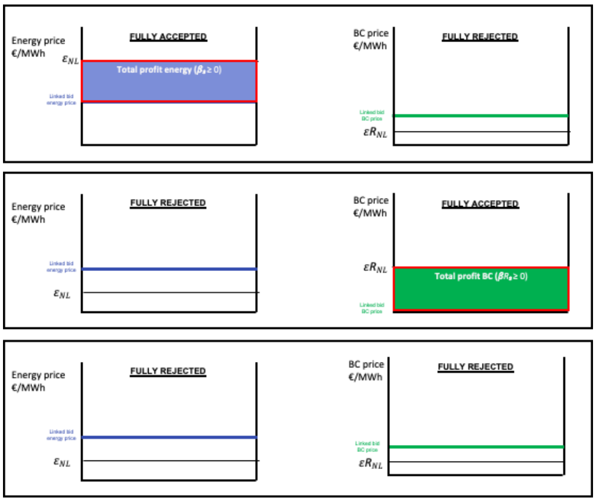
23 Even if the bid linking constraint is tight, i.e. $x_{NL}^{S} + xR_{NL}^{S} = 1$, it is not binding in the sense that removing the constraint from the problem would not increase the welfare objective function, because in case at least one of the linked bid is not profitable, this bid is not accepted anyway, even without the bid linking constraint.

24 And that consequently it cannot be the case – at least in convex problems – that a less profitable linked bid is selected because it would "create more welfare somewhere else" as has been sometimes suggested in the past.

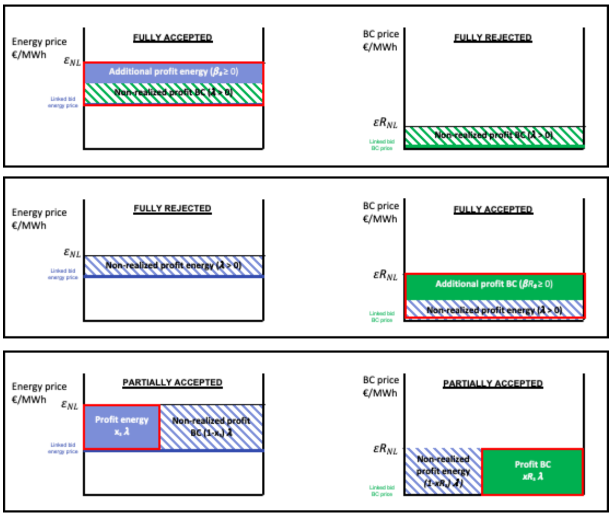
- R8: The loss of profit of the rejected linked bid is defined by λ, and the profit of the accepted linked bid is the sum of λ and an additional non-negative β or βR term. This means that **the profit of the accepted linked bid always at least compensates for the opportunity loss of the rejected linked bid**. This is in fact a corollary of R7. R7 & R8 ensure that the linked bids are always globally satisfied by the results: given the market clearing prices, in no case would they be more satisfied economically with another bid acceptance.
- $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) = QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL}) > 0$:
  In case the profitability of two linked bids are equal, solving (17) and (18) is straightforward: $Q_{NL}^{s}(P_{NL}^{s} - \varepsilon_{NL}) = QR_{NL}^{s}(PR_{NL}^{s} - \varepsilon R_{NL}) = \lambda$; therewith β_{NL}^{s} and βR_{NL}^{s} both have zero values and the two linked orders can be fractionally accepted as long as $x_{NL}^{s} + xR_{NL}^{s} = 1$ (which is necessary to enable a positive λ). We here somehow "loose" the R5 rule: fractionally accepted linked bids do not necessarily set the market clearing price. We nonetheless keep an important property: a linked bid that is apportioned over the two products is guaranteed to be equally profitable in both products. Given the market clearing prices, the bidder of such a linked bid remains fully satisfied with the outcome, despite the "tie" in the bids profitability, as its profit equals λ in such a case.

- We thus developed some additional uniform market clearing rules, for the specific case of 2 linked bids:

R6. If at least one of the two linked bids is not in the money, then R1-R5 rules apply in full.



R7. If the two linked bids are in the money, then the most profitable one is selected. Only in case of tie, the bid linking constraint can be apportioned between the linked bids.



R8. If a linked bid is selected, then its profit is always at least sufficient to compensate for a loss of opportunity of the rejected linked bid. Consequently, linked bid selection rule de facto embeds the "opportunity costs" of not being selected in the other market.

NB: R8 is the reasoning held to argue that the BC bids can be priced at a cost of zero in the simulations, i.e. because the opportunity cost of not being accepted in DA is embedded in the multilateral welfare co-optimization. In practice however, BC bids may suffer from other explicit or implicit costs (e.g. startup costs, opportunity cost for being prevented to trade in the intraday market, …)

Preliminary conclusions

In this appendix, we have analytically developed the (convex) welfare maximization problem of a simplified toy example and derived its dual and KKT equations in order to describe the economic properties of optimal co-optimization outcomes. The simplified example only contains a simple ATC interconnection and a simple linked bid, from which we derive "uniform market clearing price properties".

Such descriptions still need to be generalized to more advanced configurations but give an intuition about what to expect and how to interpret energy and balancing capacity co-optimization market coupling results.

The main objective of this document has been to highlight the fundamental relations between order acceptance and resulting clearing prices, which – when derived from duality theory – are natural outputs of resolving a welfare maximization problem (irrespective of the actual technique to resolve it – being EUPHEMIA or not) which make complete sense from an economic coherence perspective. In other words, the prices deducted from the dual variables are prices that fully satisfy all stakeholders and are economically interpretable.

For a given optimal welfare maximization bid acceptance, calculating prices which do not respect such rules is obviously possible. The welfare with such alternative prices is not modified (because the primal problem isn't modified). However, the repartition of this welfare among the different stakeholders is necessarily impacted, i.e. the surpluses of the accepted bids

(+ opportunity losses of rejected bids) and the congestion revenues become different if prices are changed, and it is no longer guaranteed that all stakeholders are “satisfied” with the results.

We also remind that in some cases, there exist multiple solutions that satisfy these equations. If this is the case for the primal problem, we refer to “volume indeterminacies” which can be graphically represented by horizontal overlaps of the supply and demand curves (i.e. all solutions on these overlapping curves provide the same welfare, even though they have different cleared volumes). Similarly, if this is the case for the dual problem, we refer to “price indeterminacies” which can be graphically represented by vertical overlaps of the supply and demand curves (i.e. all solutions on these overlapping curves provide the same total “surplus”, even though the repartition of the surplus between the buyers, sellers and grid operators may vary depending on the solution which is picked). Addressing such indeterminations is typically made separately. In case the welfare optimization problem is resolved in “calculation steps”, all the primal-dual relations are no longer naturally guaranteed. In particular, relations which link BC and energy prices are no longer enforced in a potential “second step” of a 2-step calculation, which potentially gives room to more “price indeterminacies” than in the previous optimization steps. This is the reason why even simple instances, which are perfectly solved at optimality, can suffer from “price discrepancies” in case of multi-steps calculations. By “price discrepancies”, we mean prices that do not perfectly satisfy all stakeholders.

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